GRAPHIC METHODS FOR PRESENTING BUSINESS STATISTICS

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FOR PRESENTING

BUSINESS STATISTICS

BY

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PREFACE

The fundamental principles of graphic presentation of business statistics are simple to those who will give them serious study. These principles are fairly well standardized, and, once understood, there will be no difficulty in applying them to any particular problem. A definite practice of business charting has been developed which is distinct from engineering charting though it is closely related to it. It is the object of this book to contribute to the explanation of the principles of business charting and to standardization of method and form. The discussion is confined to the most commonly used forms of statistical charts, and does not include devices like organization charts, routing charts, and other charts in which the subject matter is not statistical.

As it is assumed that the reader has a general appreciation of the use of charts, but little space is given to explaining the merits of graphic presentation as a whole. The merits of specific types and forms, however, are carefully considered. Every essential point made in the text is illustrated by an example. As far as possible the practical use of each form of chart is shown by using illustrations drawn from actual practice, though in many cases it was necessary to redraw the chart and conceal the identity of the source because of the confidential nature of the data presented.

Although the text is technical, no technical language has been used which is not fully explained. Mathematical terms which might not be familiar to the business student have been carefully avoided. Based upon extensive experience gained in explaining graphic methods to those who were not mathematicians or engineers, the author has endeavored to use a language that can be understood clearly by any businessman or business student who will give it serious study.

This book may be said to be largely a compilation of points found by experience to be necessary to an understanding of graphic methods. During the time that the author was an instructor in the Harvard Graduate School of Business Administration he gave considerable attention to the graphic presentation

part of the course in Business Statistics. In going through this work with twelve different laboratory classes, a careful record was made of the points that had to be emphasized in the lectures in order to get the best results in making and understanding statistical charts. This work was continued in university teaching in southern California, and it has been revised and added to considerably in the author's consulting practice, government research, and other statistical work.

The author wishes to express his appreciation of the help of the many persons who have contributed to the production of this volume. He wishes to thank in particular Dr. Edmund E. Day, who suggested the plan for organizing the material, liberally supplied illustrations from his files, and made many valuable criticisms of the original manuscript, and the late Col. M. C. Rorty, who wrote the introduction and made many important improvements throughout the book. He again wishes to thank Messrs. George J. Eberle, A. J. Hettinger, Thurston H. Ross, William F. French, R. S. Parker, H. W. Smith, John L. Niceley, Charles E. Berry, and William M. Holmes for valuable assistance in preparing the first edition. Deep appreciation should also be expressed to Arthur B. Fridinger, Lloyd Rogers, N. Eckhardt, Jr., Thomas P. Kelly, Ernest F. Guilford, Alfred S. Myers, R. Von Huhn, and Alice Robinson Griffith for valuable suggestions and assistance in preparing the second edition.

The author also wishes to express appreciation to the many persons, business organizations, government departments, and others who have so liberally furnished charts and other materials for use in this book. While the author has endeavored to give full credit for all quotations, material, illustrations, data, and ideas, it is impossible to acknowledge adequately all the assistance that has been received. In many instances, further acknowledgments are made in connection with the exhibits or in footnotes.

The author will welcome any critical comments on this volume, and any suggestions indicating how graphic practice can be further standardized will be especially appreciated.

JOHN R. RIGGLEMAN.

WASHINGTON, D. C.,
February, 1936.

CONTENTS

PAGE

CHAPTER VIII

CHAPTER IX

APPENDIX

INTRODUCTION

An eminent scientist has defined language as "any code of signals capable of communicating information or thought." On this basis he has asserted that a language might be devised in terms of any one of the physical senses. The language of touch might be nothing more than the Morse telegraph code transmitted by means of a series of finger taps; or it might be the embossed type used by the blind. The languages of taste and smell might equally well be constructed in the form of arbitrary codes of alternating and successive flavors and odors. And the languages of sight and hearing, as our everyday experience tells us, are nothing more than arbitrary codes of letters and sounds.

In whatever manner language has developed, it has gained in breadth and flexibility and comprehensiveness as it has become more arbitrary in form—but, at the same time, it has lost in vividness. The upraised hand of the traffic policeman speaks in a language clearer even than that which he uses when his signal is disregarded. A hand clasp, a smile, or a frown may say more than any written page.

The outstanding factor in all language, in all the varied ways in which we perceive our environment and communicate with it, is the predominant influence and importance of the visual sense. The greater part of our memories, and perhaps 90 per cent of our constructive thinking, is in terms of visual images. The man who can most successfully plan a campaign for the future is that one who can most clearly set the stage in his own mind for the play that is contemplated, and can make his actors carry out the phantom drama to the end without missing an essential move or gesture. Those who have this power of exact thinking in terms of visual images are usually unconscious of the methods by which they arrive at their conclusions. Many great mathematicians and physicists work out rough solutions of their problems by these visual methods before they apply the final check of

mathematical formulas; but they rarely can describe the type of imagery which they employ.

This general unconsciousness of the extent to which thought is expressed in terms of visual images is, however, the best of evidence as to the pervasiveness of the sense of sight in our mental processes. We *grasp* a simile to *strike* the imagination of a reader until his thoughts *run* with ours, and each word italicized represents a primitive visual image stolen from its original sense for a broader use.

With this fundamental background in human habits and experience, no apology is necessary for the attention which has been given in recent years to the study of graphics as an aid to statistical presentation. The old facts of business and human experience are valueless except as they lead to new action, sanely conceived. Such facts must, in the main, continue to be presented on the printed page—but on such pages the cold figures of a balance sheet, or the massive columns of a census report, speak in clear tones only to the expert who has the time to read them with care. For the ordinary business man, or even for the busy expert, the typed page must, therefore, have its supplemental language—a language which will help to bridge the gap between the arbitrary typed symbols of words and figures, and those mental images and that clearness of comprehension which must be created before action can ensue. This supplemental language is the language of graphics—and the essential and somewhat unique thing that Mr. Riggleman has done is to teach the language of graphics without at the same time unnecessarily confusing his reader by attempting to teach the science of statistics.

The distinction between the language of graphics and the science of statistics is as well marked as that between the mere ability to read and write and the special skill of a trained journalist. And just as the ultimate star reporter must begin his career by learning the alphabet, the vocabulary, and the grammar of the language in which he is to write, so also there is much advantage for the beginner in statistics, and in many cases for the trained or naturally gifted statistician, in studying the alphabet and grammar of graphics as a thing apart from the elaborations of statistical technique.

Mr. Riggleman has also done another thing which is worthy of remark. He has built his book out of copious notes drawn from

practical experience in the teaching of graphical methods to both college students and business men. This is the surest guarantee of the practical usefulness of the volume.

To say this much of graphical presentation in general, and of Mr. Riggleman's book in particular, is not difficult, but it is less easy to accede to the request that the writer should illustrate his point of view by specific suggestions arising from his own experience. There is little that can properly be included in the language of graphics which the present text has left uncovered. Any amplification of the subject must, therefore, be in the direction of indicating the next steps that naturally will be taken by the student after he has mastered the language. In this respect, perhaps, the greatest principle in business statistics is that every single presentation to the executive, whether graphical or otherwise, should carry its definite message. This message may be "rest content with the situation as it is"; or it may be "investigate in further detail"; or it may be a call for definite action. But in every case some message must be evident. There is no greater confession of weakness in business statistics than for the executive to say, "Very interesting—but what shall we do about it?" and for the statistician to answer, "I do not know."

A second important principle is that of approaching a difficult or confused problem piecemeal, and step by step, without insisting that the entire method of solution shall be evident from the beginning. In such cases it is always possible to select one or more facts, or whole series of data, which must obviously be assembled before any answer is possible. When these facts have been gathered together, plotted and analyzed, they will quite certainly suggest the need for further facts. And, sooner or later, if this process is repeated, the direct road to a solution will present itself.

The two principles just stated are, perhaps, the most general that apply in business statistics. For this reason it may not be inappropriate to mention them as an introduction to a volume which is characterized by the simplicity and directness of purpose that are so evident in the pages that follow.

M. C. RORTY.

GRAPHIC METHODS FOR PRESENTING BUSINESS STATISTICS

CHAPTER I

GRAPHIC CHARTS IN BUSINESS

In any well-managed business enterprise, and particularly in a large one, there is, and must be, consciously or unconsciously, a very great dependence upon statistics. And as business becomes more complex and definite planning of operations and policies becomes more essential, greater use of actual statistical facts must supersede much of the guesswork that now too commonly constitutes the bases of executive decisions. The average business man, however, finds it difficult, if not impossible, to read or analyze columns of figures and masses of data rapidly enough to base his decisions upon them when, as is ordinarily the case, he is forced to decide quickly. Fortunately, pages of cold figures can be made to speak in clearer tones to the busy executive if translated into the "graphic language." Realization of this fact has resulted in widespread use of statistical charts, especially in recent years.

Millions of dollars are spent annually in the collection of data to be used in analyzing business. Usually, however, when the measurements have been made and recorded it is difficult to appreciate the relationships. Even when data have been carefully studied it may be almost impossible to show that the conclusions reached are correct and sound. It often happens in a meeting of a committee or board of directors that a sound and carefully prepared plan is rejected simply because the facts presented are not clear. Though it is important to have accurate and significant data, one cannot get the most valuable results unless the methods of presentation are effective.

When statistical magnitudes, changes, or distributions are presented in tables or written description, it is not easy for the average mind to grasp cr appreciate the relationships and their

significances. However, by taking advantage of the condition that any business fact that can be measured can be presented graphically by means of diagrams or charts, we are able to appeal to the mind through the eye, through which medium man easily compares, either consciously or subconsciously, lengths, areas, and volumes.

It is a fact, common to the experience of everyone, that if we wish to get a conception of the size of a certain object we can do so much more easily if we can make a direct visual comparison of the unknown magnitude with a known magnitude. Thus, it is much easier to estimate the height of a wall if a man of known height stands beside it. Based on the same principle, an impression of this comparison may be conveyed to another person by means of a picture of the objects viewed. (This method is illustrated by Exhibit 10 on page 18, which indicates clearly the great size of the *Aquitania* by picturing an eight-car train upon her deck.) Nearly everyone has a good idea of the size of a passenger train and can picture it in his mind's eye.

Not only are graphic charts valuable in presenting business statistics for current use but they are valuable aids to the memory. The great majority of persons can remember the shape of a curve much more easily than they can the variation in a column of statistical data. The shape of a curve will give directly a mental picture of the changes in, for instance, sales, production, cost of living, or income over a certain period, and these visual impressions are more clearly remembered by most persons than actual figures.

One of the greatest values of the graphic chart is its use in the analysis of a problem. Invariably a chart brings up many questions which require careful consideration and often further research before a satisfactory conclusion can be reached. A properly drawn chart gives us a comprehensive picture of the situation. It will not bring out new facts, but it will bring out hidden facts.

While charts bring out the hidden facts in a table or mass of data, they cannot take the place of careful analysis. In fact, charts may be dangerous devices when in the hands of those unable or unwilling to base their interpretations upon careful study. This, however, does not detract from their value when properly used as aids in solving statistical problems.

Engineers and technical men have long realized the importance of graphic charts, and in recent years their value has been widely recognized by business executives, salesmen, advertising men, bank officials, lawyers, and others. Graphic charts have often been thought to be the tools of those alone who are skilled in mathematics, but one needs to have only a knowledge of eighth-grade arithmetic or decimal fractions to make and use intelligently even the logarithmic or ratio chart which is considered so formidable by those unfamiliar with it.

In many up-to-date plants at the present time executives and managers value their charts as highly as engineers value their drawings. Daily reports are presented in graphic form for comparison, and at a glance one can appreciate the state of affairs in his business. To most persons columns of figures are dry and uninteresting even though their study is vital to the success of the firm. What the executive desires is a presentation of this material in a form that will permit him to grasp easily and quickly the essential points, and graphic charts are a means to this end.

Graphic methods are used for presenting all kinds of statistical data. They may be used extensively as part of a well-developed statistical department or to a limited extent where a few important facts are recorded by a bright clerk or secretary. Charts may be kept of the internal conditions of a firm, and along with these the executive may directly compare external conditions with the situation in his own business. Exhibit 1, as well as many other exhibits in this book, presents some examples of actual uses by business firms.

Graphic charts furnish an excellent means for keeping a perpetual inventory which will show at a glance the value or amount of material on hand, how much is added and taken away, and when the danger point is reached. The credit manager finds charts valuable in recording a customer's condition, especially his ratio of liquid assets to current liabilities with reference to their position in the business cycle. Working hand in hand with the credit manager, the collection manager can determine at a glance the financial standing of the customer and whether or not it is possible for him to live up to the terms he has made with the company. In studying costs there is a great field for graphic charts. In fact, it is in this field that charts present some of their most surprising and important

revelations. There is almost no limit to the use of charts in analyzing labor statistics such as labor turnover, time and motion studies, causes of accident, or payment plans. In the

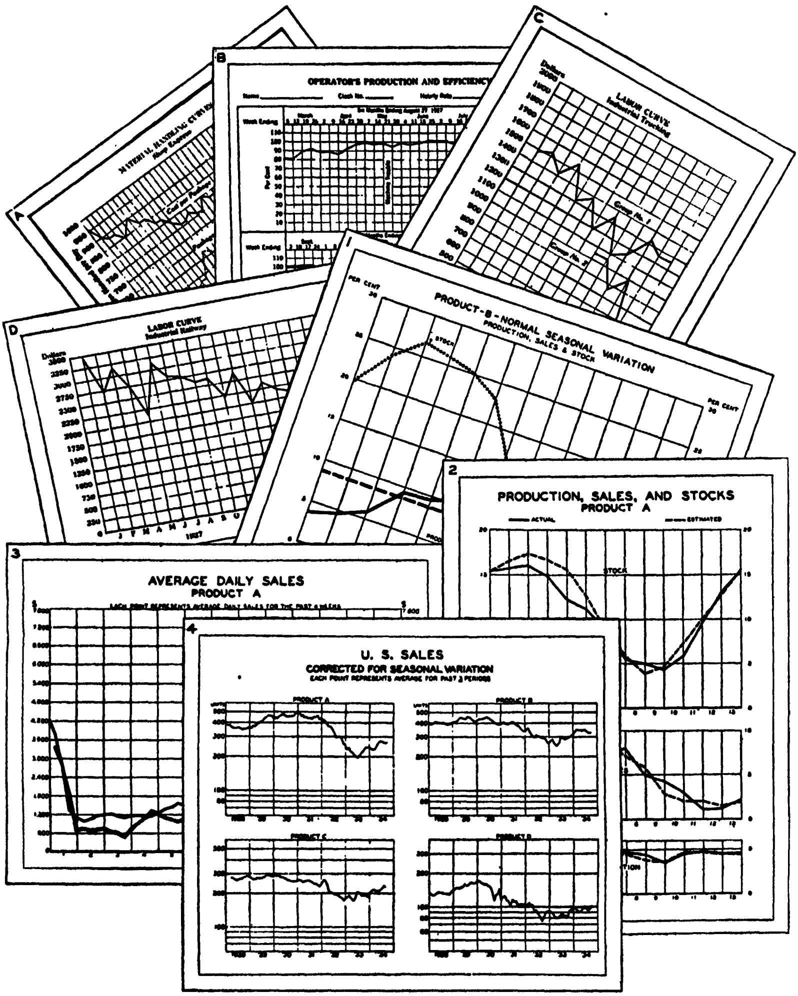


Exhibit 1.—Charts of internal statistics used by Western Electric Company (*A*, *B*, *C*, and *D*) and Eastman Kodak Company (*1*, *2*, *3*, and *4*).

sales and purchasing departments, charts are used to keep the manager in touch with the progress of events both inside and outside the firm. The study of financial statistics is greatly aided by charting the data to show amounts and rates of change.

One of the most important uses of the graphic chart is in forecasting earnings, expenses, sales, production, etc., for periods of from 1 to 5 years (revised periodically), and then in comparing actual and forecast results. The graphic method used in this way, in connection with each element of the business as well as the business as a whole, is one of the most effective of all methods of supervision.

The preceding uses and many others will be suggested to the reader by the occasions which arise in business practice if the fundamentals presented in this book are carefully studied and assimilated.

If graphic methods are to be most effective, it is necessary that those to whom they are unfamiliar give some attention to their fundamental structure. As simple as they are, their use and interpretation will be weak and often incorrect unless one understands the principles upon which they are based. For instance, one is not likely to read an arithmetic chart correctly unless he also appreciates the significance of the logarithmic chart. To explain the principles underlying the construction and use of the commonly used practical types of business-statistics charts is the object of the chapters which follow.

CHAPTER II

PRINCIPLES OF CHART PLANNING

Careful planning should precede the actual drawing of any particular chart. First, one must accurately determine the purpose of the chart. Then one must select the proper type of chart and decide upon the proportions, form, title, etc., that will best carry out the purpose. To consider these is the object of this chapter.

Purpose of the Chart.—Two broad groups of graphic charts, namely, working charts and finished charts, should be distinguished clearly. Working charts are usually made on coordinate paper and are unfinished in appearance. Finished charts are ordinarily drawn upon plain drawing paper by a skilled draftsman with the aid of drawing instruments.[1] The statistician, in making his analyses, uses many working charts which may be of value only to him and which often have only a temporary or transitional value. Such charts are not ordinarily carried beyond the stage of pencil plotting on coordinate paper. But if the chart is to appear in a formal report or printed publication, a finished drawing should be prepared by a skilled statistical draftsman. Before proceeding with the work on a finished drawing, one or more working charts ordinarily should be

[1] It is assumed in this book that the reader has a working knowledge of the use of the usual mechanical drawing instruments and that he has studied lettering. However, for the use of those who may not have had such training, and as an aid to those who are interested in special statistical drafting practices and short cuts, an appendix on statistical drawing and lettering is provided on pp. 217–244. For the reason that the beginning statistician can often use his knowledge of statistical drafting as a means of securing a position, careful attention to the details discussed in this appendix is strongly advised. A technical knowledge of the actual use of instruments and lettering is also very helpful to the experienced statistician or executive in supervising chart work and in judging its quality. It also helps one to read more correctly and accurately the data presented in the graphic language as it requires greater attention to certain details than is necessary in making working charts.

prepared as a basis for more accurately determining the use of the chart as well as its form, proportions, size, etc.

It will be evident from the problem at hand whether or not a working chart is required. In this discussion, principally the details of making finished charts will be described for the reason that, if these are understood, there will be no difficulty in making and using working charts.

In planning the construction of a chart for a report or a publication, a cardinal principle to be remembered is that the reader's point of view must always be kept in mind. When there is a question as to which of several methods should be used, that method should be chosen which will enable the reader to grasp the essential points the most easily and clearly. It often happens that the chart which is easy to make is difficult to read, and that the chart which is easy to read is difficult to make. However, since the chart is made for the reader rather than for the maker, the decision on any point should be in favor of the reader.

When charts are prepared for the use of the skilled executive, and at his request, it is to be presumed that he knows enough of, or will make an effort to learn enough of, the technical principles of the graphic language to enable him to read his charts effectively. But when the purpose of a chart is to make an appeal to the casual or untrained reader, simple methods, which are already familiar to the person appealed to, must be used. Plain technical graphs will be used by the executive or the board of directors who already have an interest in the subject matter to be presented. On the other hand, charts for the untrained reader may have to be in some popularized form such as those in which pictures or cartoons help to attract attention and bring out the facts represented. Charts will vary all the way from popularized cartoons representing simple sizes to technical scientific graphs, depending upon their purposes and the training of the prospective reader. In popular periodicals the tendency naturally is toward the use of simple diagrams, while in technical publications there is an advantage in using the customary technical forms.

Size and Proportions of the Chart.—As mentioned previously, charts are usually first made in the form of working charts, especially if they are complicated. Before starting work on a

finished chart that is to be made from a working chart, it is often well to outline it first as a rough draft in order that the best proportions may be determined for the final drawing.

The size of a chart should be determined on the basis of convenience in handling and ease of reading. It is often best to have several small charts instead of a large one, especially if this large chart has to be folded. A folded chart is always difficult to handle and is easily damaged. There is a tendency to make charts larger than necessary. As a general rule, however, it is best to use the smallest size that can be read conveniently. The larger the number of charts that can be placed upon one page, without making them so small that they are hard to read, the greater will be the convenience in using them.

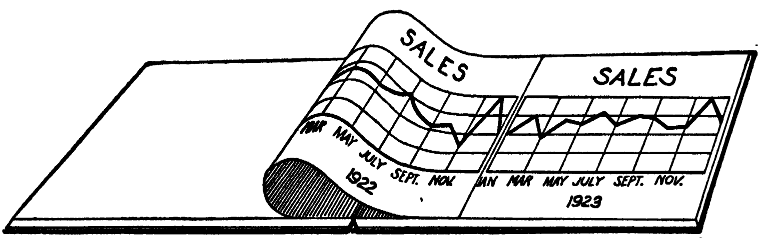


EXHIBIT 2.—Consecutive pages forming a continuous chart.

An exceptionally long chart—for instance, one presenting a time series plotted by months over a period of 20 years—may be made more convenient to handle if it is broken up into, say, 20 charts and bound together in book form. The consecutive pages then form a continuous chart across the break between pages when used as illustrated in Exhibit 2.

In making a large chart, such as those in Exhibit 3, it is advisable to lay it out in detail in small form and then enlarge it to the desired size. For instance, a wall chart might first be laid out 6 by 12 inches and then enlarged to 3 by 6 feet, all dimensions, widths of lines, lettering, etc., being enlarged exactly six times (linear). The small chart should, of course, be carefully checked for accuracy before it is enlarged.

Charts may be enlarged by a draftsman by hand or by photographic processes. If enlarged by a draftsman, they are usually drawn in pencil on heavy paper and traced on tracing cloth or paper, from which a Van Dyke and brown- or blue-line prints are

made. Often the most satisfactory and least expensive method of making wall charts is to enlarge skillfully drawn small charts by photographic methods. The best photographic process is to make a small plate with a camera, and from this plate to make an enlargement of the desired size on a special thin paper. From this enlargement a Van Dyke can be printed and then any number of brown- or blue-line prints can be made. Satisfactory enlargements are also made by the photostat method. (For a description of brown- and blue-line printing and the photostat process, see Chapter IX.)

EXHIBIT 3.—Wall charts of statistical data used for exhibition purposes. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

Use of Printed Forms.—Various kinds of ready-ruled forms and coordinate paper can be used in charting business statistics. Some examples of such forms are illustrated in Exhibit 4. These forms are convenient for preliminary graphs and working charts but they are rarely suitable for finished work, such as that prepared for formal reports or publication. Even in making working charts, it is often easier to draw a special form to fit the problem than it is to fit the problem to a ready-ruled form.

On finished charts, no more coordinate lines should be shown than are necessary for the purpose of reading to the required degree of accuracy. It very rarely happens that standard coordinate paper is so ruled that it has the desired divisions, or

that a chart made on it presents a good appearance when it is finished. When charts can be standardized, it is frequently desirable to make up zinc plates or to use the "offset" printing process in order to secure just the kind of coordinate paper required. In some cases permanent titles, scales, etc., can be

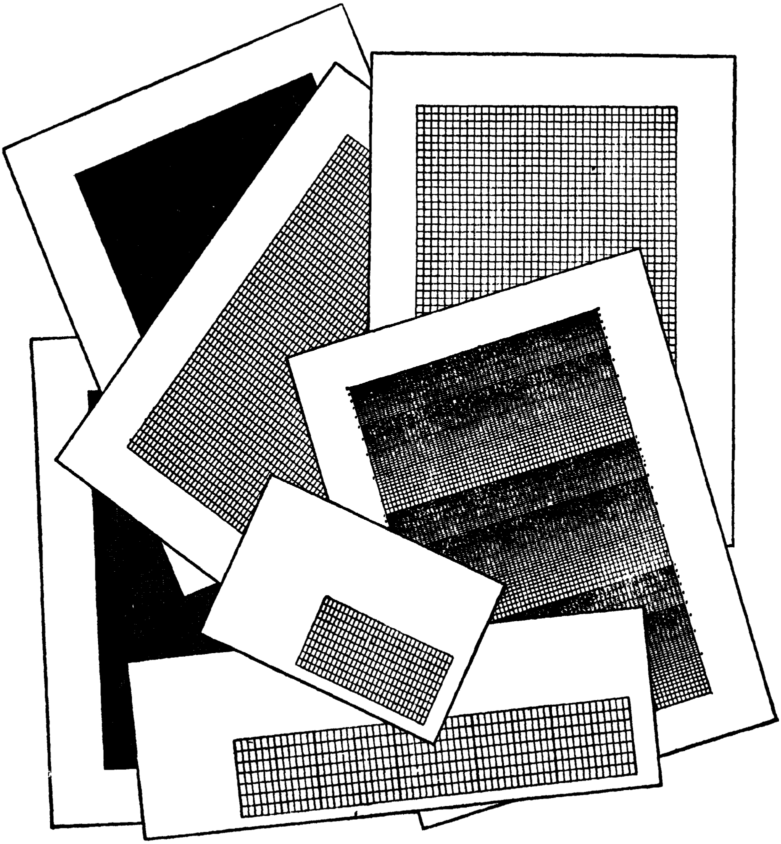
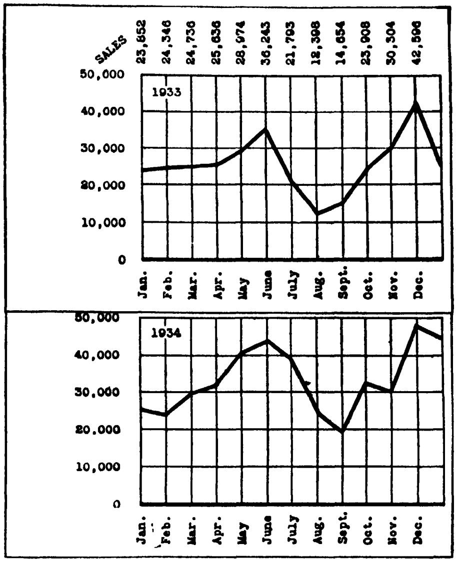


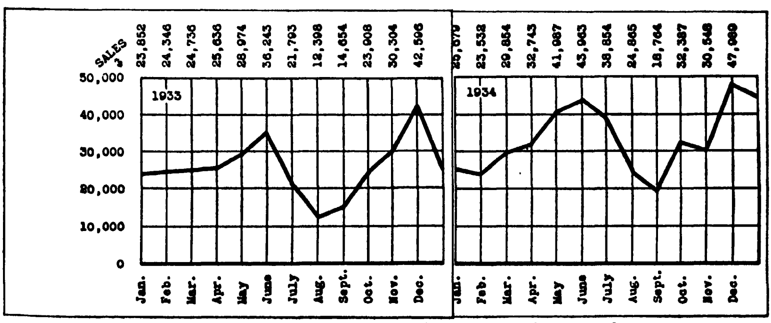
EXHIBIT 4.—Coordinate paper for preliminary and working charts. (*Courtesy of Educational Exhibition Co.*)

printed on the original paper. If the coordinate lines are properly spaced, accurate plotting can be done between them by using the right decimal scales, by using the scale as illustrated in Exhibit 229 on page 223, or by making special scales out of thin strips of celluloid, marking the divisions with a knife-edge and filling the grooves with black ink.

Standardization of Charts.—A considerable amount of time and labor can be saved by standardizing graphic forms where possible. By using charts of the same size and which have the



A. Cards placed one above the other for comparing seasonal variations.



B. Cards placed end to end to form a continuous chart.

EXHIBIT 5.—Standardized card records. (Also illustration of use of typewriter lettering.)

same scale intervals, comparisons can be made between different charts as illustrated in Exhibit 5, *A* and *B*. With the charts arranged vertically, one above the other, a direct comparison of seasonal changes can be made, and when arranged horizontally,

or end to end, it is easy to see the changes that have occurred in the curve throughout the period as a whole.

Accuracy in Charting.—In charting business statistics a reasonable degree of accuracy is, of course, necessary, but the problem differs from that of plotting data where measurements are made from the charts with finely graduated instruments. If the chart is accurate to the degree that visual comparisons will not be affected, it is accurate enough for most practical purposes. Care should be taken, however, to make charts accurate to this degree. Carelessness in plotting or checking may cause charts to be very misleading.

Reference to Source of Data.—The source of the data presented should ordinarily be included on a chart if the source is not evident from the title or the nature of the data. Knowledge of the source helps the reader to judge the reliability of the chart, to check it for accuracy, and to obtain further data in connection with the subject represented.

It is sometimes desirable to include the data in tabular form along with the chart, either as part of the chart or as an adjoining table. This plan is especially valuable to readers who wish to make use of numerical data in connection with the graphic method (see Exhibits 33, 34, 91, and 92, on pages 31, 32, and 78, for illustrations of recording data directly on charts).

Importance of a Clear Title.—One of the most important parts of a business chart is its title. The title should give the reader the subject of the chart clearly and completely. Ordinarily the title should tell *what* is represented and *where* and *when* the data apply (Exhibit 6). Often the title is given too little attention, and though the chart may be very clear in its other parts, it may not be read at all unless it has a clear and complete title.

If possible, the title should be short, but it may be long if necessary for the sake of clearness. It may consist of a major title with a subtitle which adds further explanation to the main subject (see Exhibit 39 on page 36 and Exhibit 123 on page 105). It is sometimes difficult to avoid technical words, and here, as in the rest of the chart, the decision must be made with due regard for the point of view of the reader. Superfluous wording should not be used as, for instance, in the title "Chart Showing Curve of Jones Brothers' Sales for 1935." Here the first four

words are unnecessary, as anyone would recognize a chart showing a curve. A better title would be simply "Jones Brothers' Sales for 1935."

Titles are usually in the form of a series of phrases rather than complete sentences (see Exhibits 39 and 40 on pages 36 and 37), and the arrangement of these phrases is such that they are easy to read and the important items stand out prominently. Each line of phrases represents an idea or group of ideas. The lines, usually of unequal lengths, are centered on the chart.

The preceding discussion of chart titles applies to finished charts, such as those for formal reports or publication. Such

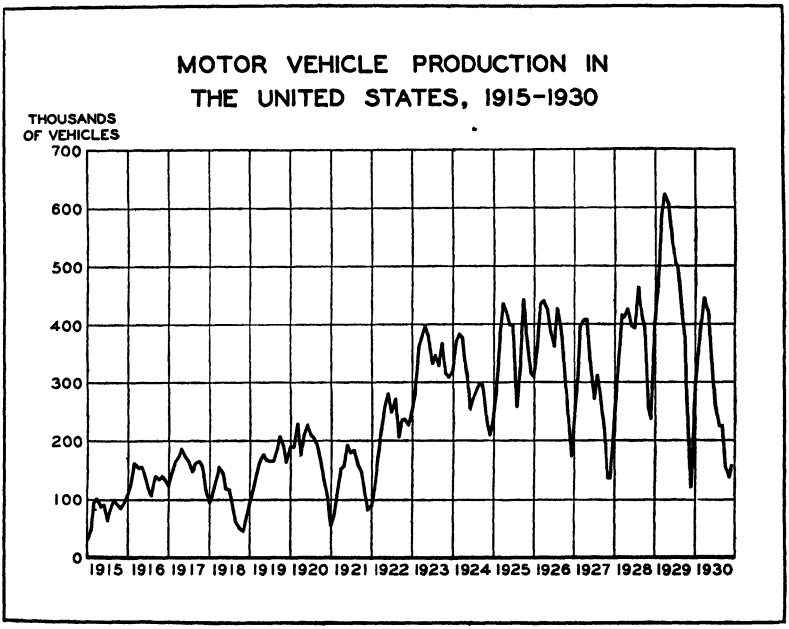


EXHIBIT 6.—Correct position for a chart title. (Chart also illustrates centering of parts of chart with reference to border lines.)

charts are usually prepared with a wide range of readers in mind. On working charts, the titles and other means of identification need be clear only to those who are concerned with using them, and, consequently, such titles may be more limited than those prepared for wider distribution. Nevertheless, the titles should be complete and clear to those who are to use them, even if they are for only preliminary or trial graphs. One of the most

important items that should be put on a working chart is the date of preparation. This record enables one to retrace his steps chronologically, and it is of great help in relating different charts to each other and in determining whether one supersedes another.

Position and Size of the Title.—The title should be placed at the top of the chart as illustrated in Exhibit 6. This position is convenient and natural from the point of view of the business man, since his bills, tables, books, accounts, schedules, and other forms are commonly labeled at the top.

No general rule can be laid down for determining the size of the title lettering. Only on the basis of skill and experience can one best decide the most appropriate size for each particular problem. The title lettering should be the largest and most prominent of any on the chart, however, and by studying other charts one can soon become efficient in determining the sizes for the problems at hand.

Spacing and Arranging Different Parts of a Chart—Border Lines.—In order that a chart may appear to be well-proportioned

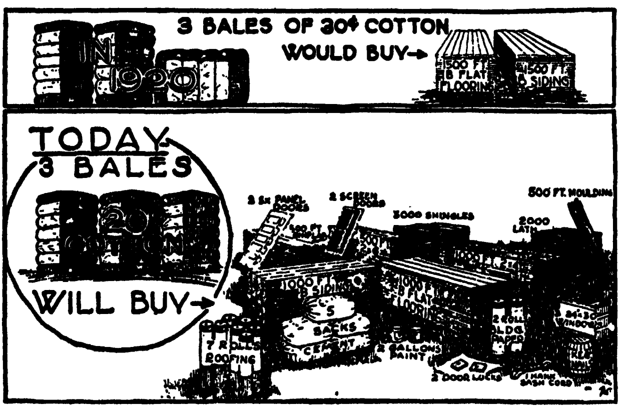


EXHIBIT 7.—Pictorial chart for popular presentation. (*Courtesy of Retail Lumberman.*)

and of pleasing symmetry, careful attention must be given to spacing and arranging its different parts. This requires that side margins, for instance, should *appear* to be equal even though they might have to be actually unequal to give the desired effect. In order to have the right and left margins appear to be equal on a graph which has a vertical scale on only

one side, less space should be allowed in the margin on the left between the scale and the border line than between the last ordinate and the border line on the right. The reason for this is that the distance between two solid lines appears to be less than an equal distance between a solid line and a scale with its

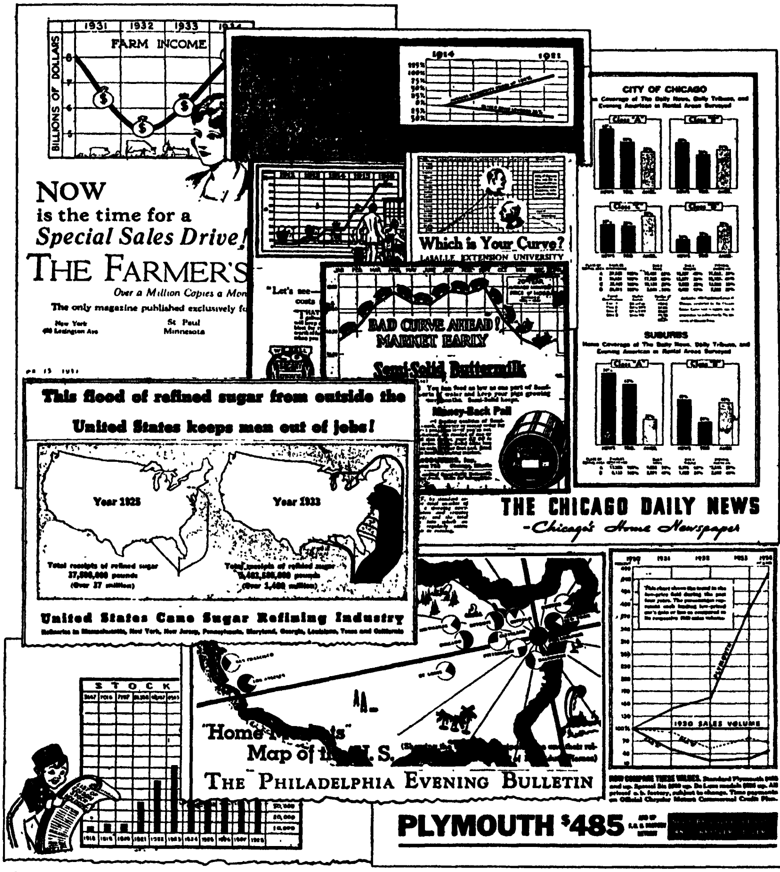


EXHIBIT 8.—Use of graphic charts in popular magazine and newspaper advertising.

open spaces. It should be noted that, in a chart like Exhibit 6, the title should be centered between the vertical border lines—not over the center of the space covered by the coordinate lines. It should also be noted that, since room must be allowed for the scale, the center between the first and last vertical lines of

the coordinate ruling of a chart like Exhibit 6 will be a little to the right of the center between the vertical border lines.

Use of Color.—Charts in which colors are used to bring out differences or contrasts are much more effective than those in which only black is used. Colors, however, are limited in their use to work in which their cost is not prohibitive. When charts are not to be reproduced, it is no more expensive to use colors than black ink alone. But if the chart is to be reproduced in a printed report, catalogue, book, or magazine, it may be impossible to use colors because of the high cost of color reproduction.

Pictorial Charts.—When prepared for popular use, charts are often much more effective if made in a pictorial form, as shown in Exhibit 7, or with pictorial additions and ornamentations which help to direct attention to the subject matter, as illustrated by some of the charts in Exhibit 8. These methods of popularizing may be applied to most of the more technical forms of business charts. A description of the methods of construction and use of the pictorial form is included in the chapter on simple comparisons of size (Chapter III).

General Appearance.—Confidence in a chart is much greater if it is neat and accurate rather than slovenly in appearance. Skill and neatness come with practice and experience. The chart must be well proportioned; it must not be crowded; and ample margins must be left. Usually a border line will improve the appearance of a chart, and it is of utmost importance, in securing an appearance of high quality, not only to have all lines and curves skillfully drawn but also to have the title and all other lettering done neatly and carefully.[1]

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[1] See Appendix on use of drawing instruments and lettering (pp. 217–244).

CHAPTER III

SIMPLE COMPARISONS OF SIZE—BAR CHARTS AND PIE CHARTS

It is the purpose in this chapter to describe the graphic methods of comparing magnitudes where only the sizes vary. There is no consideration of time variations, geographic distributions, or other variables. For instance, we might compare the populations of New York and Chicago, or the heights of buildings in New York, or the sales made by different salesmen, or the profits made on clothing with those made on shoes. Whatever the problem, the object is to compare sizes. In Exhibit 9 the problem is to show the comparative sizes and especially the heights

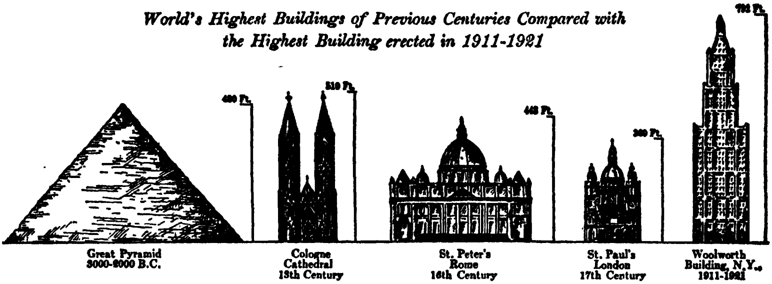
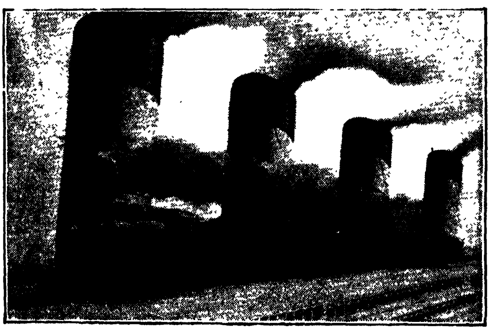


Exhibit 9.—Pictorial comparison of size (with emphasis on heights). (*Courtesy of Encyclopaedia Britannica.*)

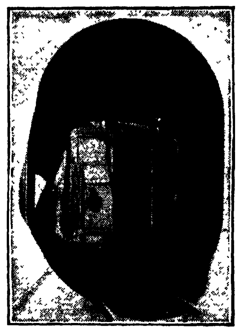
of the different structures. In other problems the object may be to indicate the relation of one size to another where both are known, or it may be to give a conception of an unknown size by comparing it with a known size. For instance, most persons are familiar with the size of a passenger train. In order to impress the greatness of the size of a modern steamship upon the readers of a popular magazine, the graphic comparisons shown in Exhibit 10 were made. Such comparisons, however, are very general and can be used only when the actual objects can be compared. In most cases of graphic presentation of sizes, the comparisons are made by using certain geometric forms which vary in their sizes in proportion to the different magnitudes represented.

Choice of Geometric Forms for Simple Comparisons of Aggregates.—When one is about to undertake the graphic comparison of simple sizes, the first point to be decided upon is the geometric form to be used. The two fundamental requirements of this form are that it must be easy to understand and that it must give a correct impression.

It is easier to compare the sizes of forms which vary in one dimension than those which vary in two or three dimensions. In other words, it is easier to compare lines or bars than it is to compare areas and especially volumes (see Exhibit 11). It would be very difficult for the average person to compare sales in New York with sales in Chicago by looking at the cubes in Exhibit 11*C* because they are drawn on a volume basis and vary



An eight-car train, with locomotive and tender, could be placed upon the upper deck of the "Aquitania" and reach less than half the ship's length.



A New York subway train could pass through a funnel of a great ocean liner with room to spare.

EXHIBIT 10.—Pictorial comparison of size. (*Courtesy of Cunard Steamship Co.*)

in three dimensions. Also it would be difficult to compare these data by use of the squares shown in Exhibit 11*B* which are drawn on an area basis and vary in two dimensions. In Exhibit 11*A* the lines or bars vary only in one dimension, and it is easy to see that New York sales are twice as large as those of Chicago. It is this form (11*A*) that is most commonly used in making graphic comparisons of size, as it is much easier to compare the lengths of bars than it is to compare areas which vary as the square of their sides, or volumes which vary as the cubes of their edges.

Although, as a rule, lines or bars should be used rather than areas, an exception to this rule may be made when showing great extremes. When it is desirable to present comparisons such as those in Exhibit 12, only a general impression of the

differences in size is ordinarily necessary, and either square or circular areas will serve the purpose with certain advantages over bars. If bars are used in making such comparisons, the larger figure requires such a long bar that it is difficult to place it on the

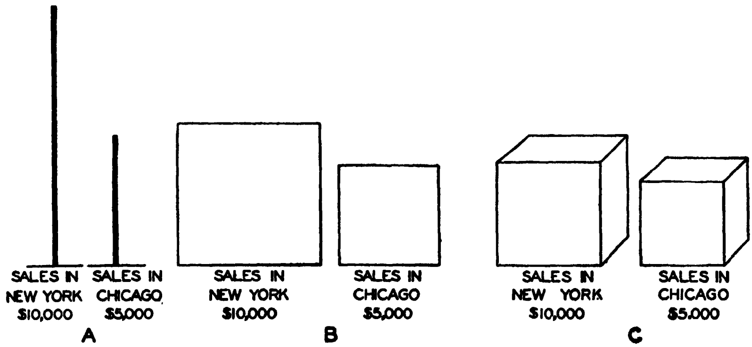


EXHIBIT 11.—Use of different geometric forms in comparing sizes (sales). *A*, Lines (or bars) are easy to compare. *B*, Areas are difficult to compare. *C*, Volumes are very difficult to compare.

page. Or if the long bar is made a convenient length, the short bar can hardly be shown at all.

Circles are used occasionally to represent sizes, but it is difficult to compare them, and they should not be used when bars can be used conveniently. When it is necessary to use circles, as, for instance, in making comparisons of extreme items (Exhibit 12) or in making certain kinds of statistical maps (Exhibit 185

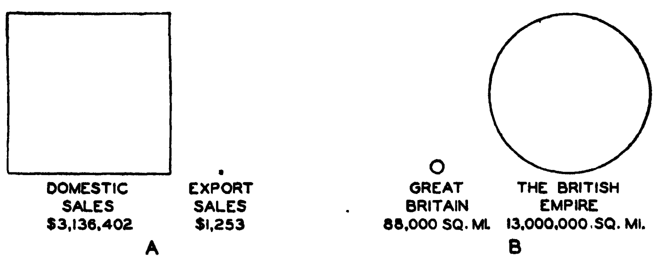


EXHIBIT 12.—Square and circular areas are sometimes advantageous when showing extreme sizes, but they should not be used when it is convenient to use bars.

on page 174), they should be drawn on an area basis rather than on a diameter basis. Sometimes, however, circles are drawn on a diameter basis and often the reader does not know which method has been used. Exhibit 13*A* compares sales in New York

with sales in Chicago by means of circles drawn with areas proportional to sales, and Exhibit 13*B* compares them with

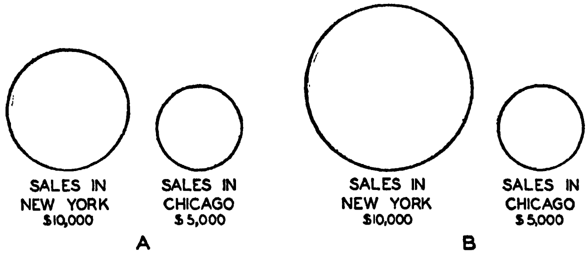


EXHIBIT 13.—*A*, Circles with areas proportional to numbers. *B*, Circles with diameters proportional to numbers.

diameters proportional to sales. Exhibit 13*A* gives the impression that New York sales are somewhat less than twice as large as Chicago sales, while Exhibit 13*B* gives the impression that New York sales are much more than twice as large as Chicago sales.

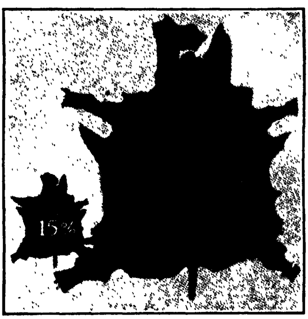


EXHIBIT 14.—Pictorial comparison of size on an area basis. (*Courtesy of Swift and Co.*)

Pictorial Forms.—The principles just discussed in connection with simple lines, areas, and volumes must be kept in mind when using pictorial forms.

It is especially difficult to compare irregular areas such as those shown in Exhibit 14. Although the method is quite effective in

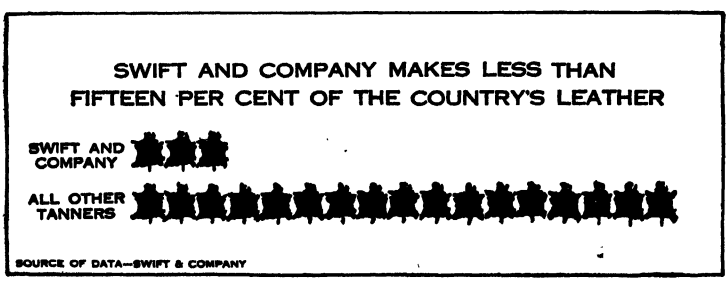


EXHIBIT 15.—Pictorial comparison of size on a line basis. Easier to read than Exhibit 14.

showing the readers of a popular magazine that Swift's tanneries handle but a small part of the total, a more accurate picture would

be given by using a chart like Exhibit 15. Swift does not tan smaller hides but a smaller number of hides. In making a popular appeal, the use of the illustrations of hides in both Exhibits 14 and 15 makes the chart much more effective than would the use of ordinary bars. It is still more difficult to make the comparison in Exhibit 16, since volumes must be considered. Here the attempt to make a striking appeal has been very successful, which probably is the principal object of

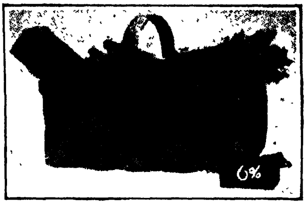


EXHIBIT 16.—A striking appeal, but difficult to compare sizes. (*Courtesy of Swift and Co.*)

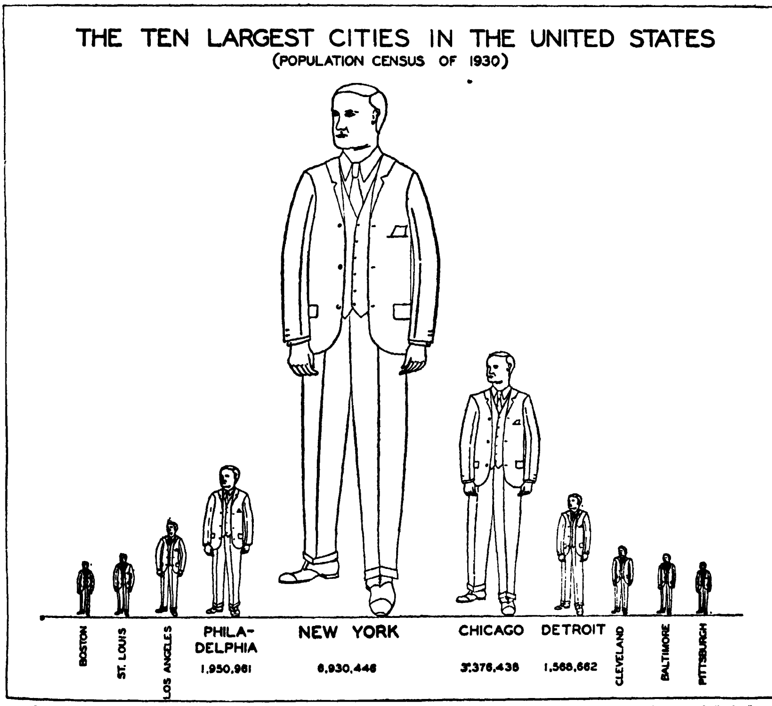


EXHIBIT 17.—A poor method of comparing the populations of cities. Heights of persons are proportional to population represented. The reader unconsciously makes the comparison on a volume basis.

the chart, but it is not easy to make an accurate comparison of size.

The purpose of Exhibit 17 is to compare sizes of cities. It is supposed to show, fori nstance, that New York is approximately twice as large as Chicago, but, as a matter of fact, the effect is that New York is much more than twice as large. The chart is so drawn that the heights of the figures are proportional to the numbers represented. Charts of this kind are quite common and usually give false impressions, as the reader compares the figures on a volume basis which greatly exaggerates the actual differences in the numbers represented. If it is desired

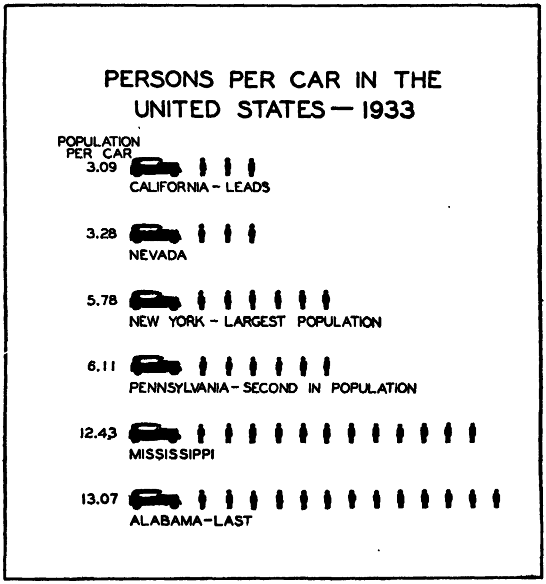


EXHIBIT 18.—Good popular presentation. Better than Exhibit 17. Comparisons are made on a line or bar basis. (*Adapted from Automobile Manufacturers Association.*)

to use the figures of persons in popular presentations of this type, it is far better to represent the numbers, not by the heights of persons, but by rows of persons evenly spaced with the lengths of the rows proportional to the numbers represented. New York does not have larger persons than Chicago, but a larger number of persons. This principle is brought out by Exhibit 18 which shows the number of persons per car for given states. The figures are equally spaced with the lengths of the rows exactly proportional to the numbers of persons represented. Another method of making a popular presentation of similar automobile

figures is illustrated by Exhibit 19. When it is desirable to use figures of persons to express numbers, methods such as those shown in Exhibits 18 and 19 should be used, for they have all of the advantages of Exhibit 17 and do not have the serious fault of leaving a very inaccurate impression.[1]

There are many pictorial figures that can be used in popular presentations, which may be made to vary in only one dimension without throwing them out of proportion, as would be the case in varying only the heights of persons. This is illustrated by

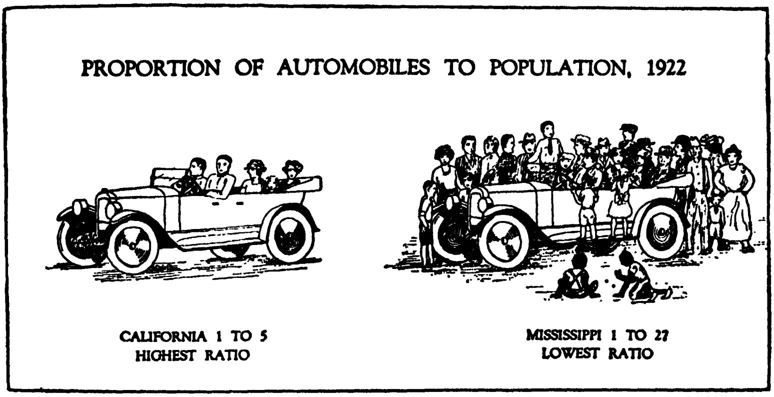


EXHIBIT 19.—Good for certain types of popular presentation.

Exhibit 20 in which height is the only dimension that changes. It is very easy to see from this chart that automobile sales are nearly twice that of each of the other three products represented. Note that the title "The Skyscraper of Industry" makes the use of buildings instead of bars particularly significant. This chart might be improved by using the same style of front for all of the buildings, since the only difference to be represented is that of size measured in terms of dollars.

There is a fairly common use of spectacular designs, such as Exhibit 21, which mean nothing in particular, as it is practically impossible to interpret them. In this instance there is little, if

[1] An extensive system of symbols, similar to those used in Exhibits 15 and 18, has been worked out by Dr. Otto Neurath, Director, International Foundation for the Promotion of Visual Education and of Mundaneum Institute (The Hague) in an attempt to develop an international picture language.

any, consideration on the part of the illustrator for an effective comparison of sizes, as the chief aim is to build up a striking architectural form.

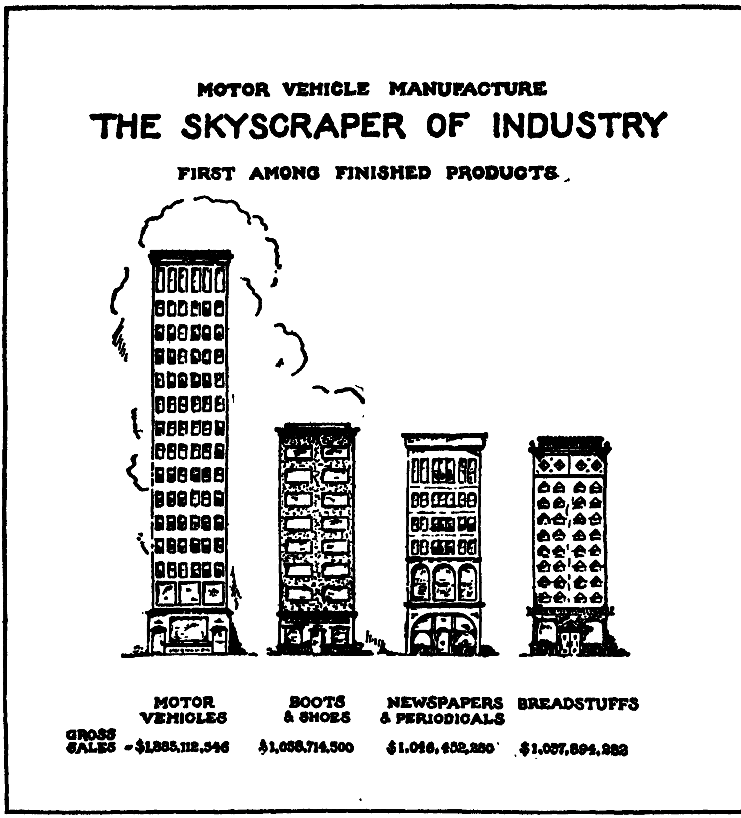


EXHIBIT 20.—Good pictorial presentation (line or bar basis). (*Courtesy of Automobile Manufacturers Association.*)

Bar Charts.—The most common method of graphically representing comparisons of size is by one form or another of the "bar chart." There are several variations of this form of chart and they have been given many fanciful names, but all are made up of vertical or horizontal bars so drawn that their lengths are proportional to the sizes represented.

For simple comparisons, charts similar to Exhibits 22 and 23 serve the purpose; but when a chart is to present several items for careful analysis, it must include certain details, such as a

clear title, a properly constructed scale, and an explanation of units used.

Vertical and Horizontal Bars.—There is no particular rule for choosing between vertical and horizontal bars, except that those

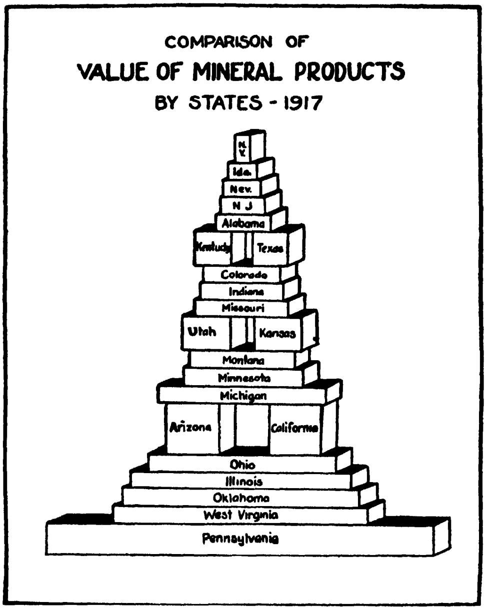


EXHIBIT 21.—Very poor presentation.

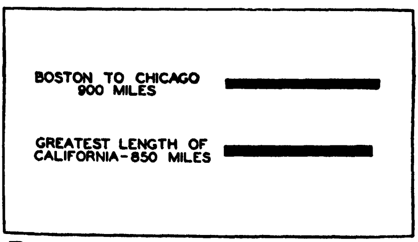


EXHIBIT 22.—Simple horizontal bar chart.

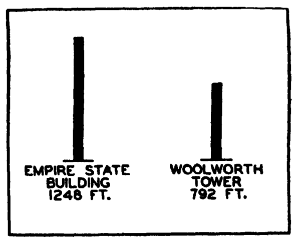


EXHIBIT 23.—Simple vertical bar chart.

should be used which will enable the reader to understand the chart the most readily.[1] In cases where the things represented

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[1] This statement applies only to simple comparisons of size. In showing time series, only vertical bars should be used, see p. 59.

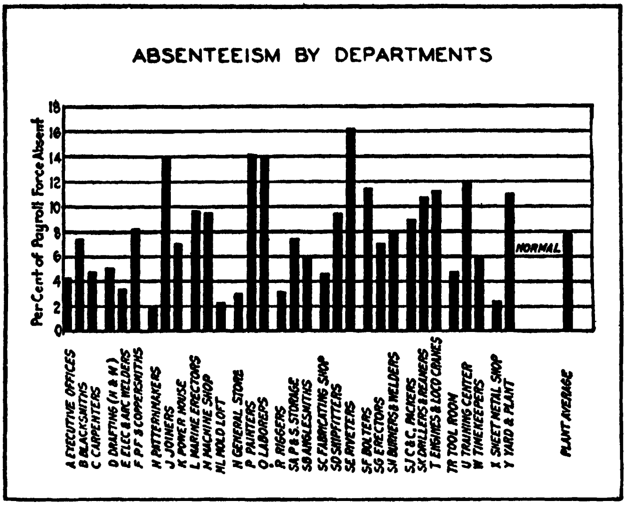

EXHIBIT 24.—Compare with Exhibit 25 for ease of reading lettering. (*Courtesy of Industrial Management.*)

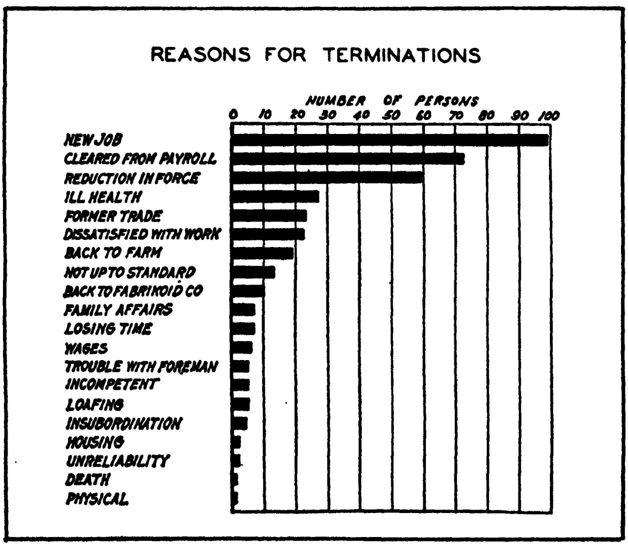

EXHIBIT 25.—Easier for the business man to read than Exhibit 24. (*Courtesy of Industrial Management.*)

vary naturally in either a horizontal or vertical direction, the bars usually are made to vary in the corresponding direction. For instance, the bars in a comparison of the distance from Boston to Chicago with the greatest length of California would naturally be made in a horizontal direction (Exhibit 22), while those in a comparison of the height of the Empire State Building with that of the Woolworth Building would be made to extend vertically (Exhibit 23). Convenience in reading the lettering is another point that must be considered when choosing between horizontal and vertical bars. If horizontal bars had been used in Exhibit 24,

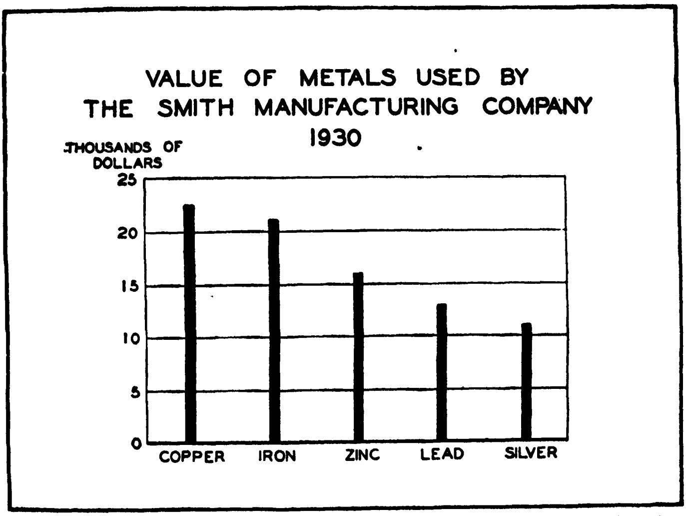

EXHIBIT 26.—Correct use of bar-chart scale. The scale should begin with zero as shown.

the designations of the bars could have been lettered horizontally, which would have made the chart much easier for the average business man to read (compare with Exhibit 25). For this reason, many statisticians prefer to use the horizontal bar chart exclusively when making simple comparisons of size.

Bar Chart Scales.—Except in cases of rough or popular comparisons, it is important to include a scale on a bar chart. A scale enables one to read and check for accuracy much more easily than when the bars are presented without one. It is a fundamental rule in making bar charts that the scale begin with zero (Exhibit 26). This rule is sometimes broken by the

inexperienced chart maker, and the values are inaccurately represented as in Exhibit 27. The bar for copper is about twice as long as the bar for silver when correctly drawn (Exhibit 26). But if the scale begins with ten instead of zero (Exhibit 27), the incorrect graphic impression is that copper is about ten times as important as silver.

Scales should be laid out in round numbers of convenient intervals. A keen sense of form, appearance, and purpose is the best guide in scale making. These will be gained with experience. The units should be so selected that the eye can read them easily and carry them across the chart with the aid of guide lines

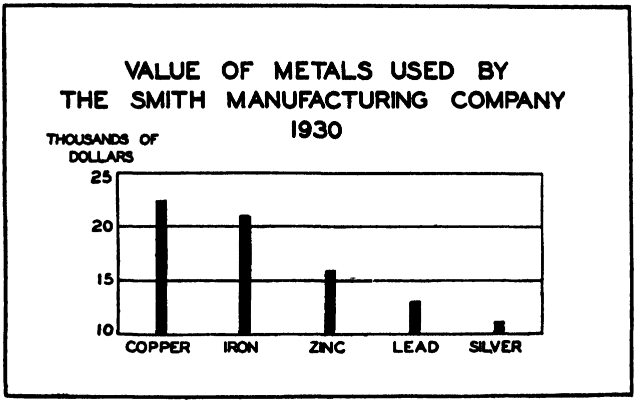

EXHIBIT 27.—Very bad practice. The scale does not begin with zero, hence the lengths of the bars do not represent the sizes accurately. (See Exhibit 26.)

(Exhibit 26). The zero lines of all charts should be made heavy so they will be sharply distinguished from the other lighter guide lines. Scales such as 0, 3, 6, 9, or 0, 7, 14 should not be used as they are hard to read. It is much easier to read a scale of units, 2's, 5's, or multiples of 10, or halves and quarters of 100, as 25, 50, 75, etc. Too many numbers in the scale are confusing and hinder rather than aid in reading the chart (Exhibit 28). It must be kept in mind that the purpose of graphic methods in business statistics is to present a picture of figures, and that it is not to make charts from which actual values are to be measured with finely graduated scales or instruments. The aim should be to construct the chart in such a way that it can be comprehended simply by visual inspection.

The commonly accepted position for the scale of a vertical bar chart is on the left and outside the background of guide lines (Exhibit 26). Each number should be exactly opposite the

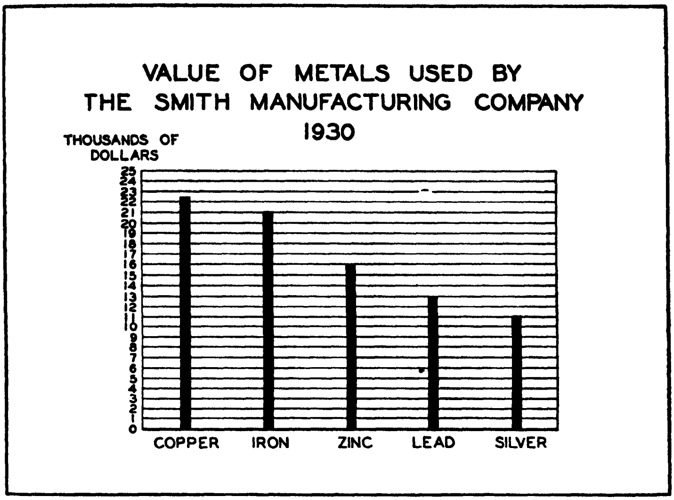

Exhibit 28.—Too many scale numbers and too many guide lines across the chart. (Compare with Exhibit 26.)

line which it represents (Exhibit 26). If it is desirable to have a scale on the right of the graph, one may be placed there in addition to the one on the left, but the left scale should in no case be omitted. The units represented by the scale should be indicated clearly by a caption lettered horizontally at the top of the column of figures (Exhibits 26 and 29*A*). This is preferable to the method of lettering vertically along the side of the column (Exhibit 29*B*), as the business reader, who often is not adept at reading engineering drawings, does not have to turn his head or the paper. Expressions such as "1 inch = \$100" are not used in statistical charting. The scale on the graph is much more convenient and furnishes all the aid necessary for measuring the different bars.

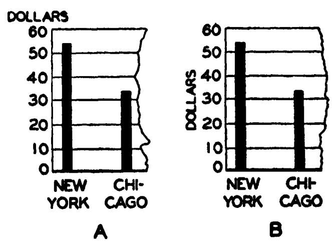

Exhibit 29.—Position of scale unit designations on vertical bar charts. *A*, Best practice for business charts. *B*, Common engineering practice.

The scale on horizontal bar charts is commonly placed along the top just above the bars (Exhibit 30). It may be placed along the bottom also if it will add anything to the clearness of the chart. The scale units of Exhibit 30 are thousands of dollars, and this fact should be indicated as shown by the designation just above the scale. If there is a scale below the chart, the designation of units would be shown, of course, just underneath the scale. Exhibit 31 illustrates a good method of including a scale when no background of guide lines is used.

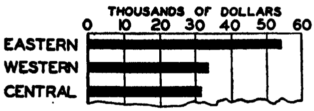

EXHIBIT 30.—Designating horizontal bar-chart scales.

Guide Lines to Aid in Reading Scales or Comparing Bars.—When making charts for careful reading, it usually is best to include a background of guide lines which assist in comparing the bars or reading from the scale (Exhibit 32). Judgment must be used in selecting the number of lines to be included on the finished chart. Too many lines are as bad as too few. Exhibit

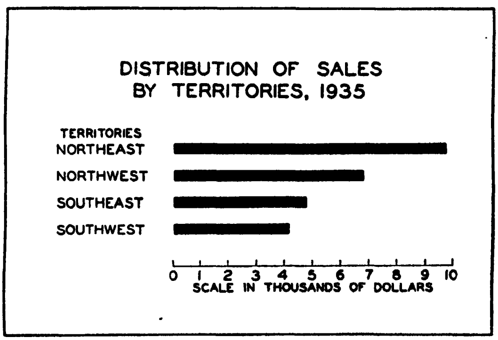

EXHIBIT 31.—A method of including a scale when no background of guide lines is used.

28 illustrates the use of too many lines (compare Exhibits 26 and 28). These lines should not be prominent and should appear to be behind the bars. That is, they should be drawn between, not through, the bars (Exhibit 32). This point is not so important, of course, in the case of solid black bars as when shaded, colored, or outlined bars are used.

Designating Bars.—Designations explaining what vertical bars represent usually are placed underneath the bars as shown in Exhibit 33. It is sometimes desirable to present statistical figures

as part of a chart, and in cases of this kind they may be placed underneath the designations, as in Exhibit 33. This position is convenient for recording and reading and in no way interferes

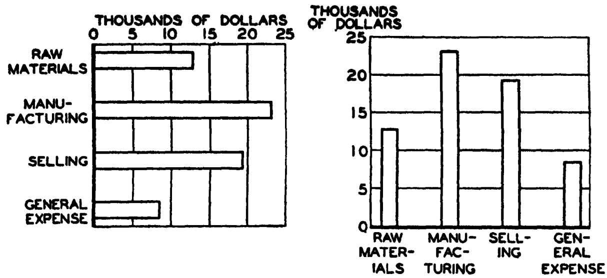

Exhibit 32.—Guide lines for reading the scale should not be drawn through outlined shaded or colored bars.

with the effect of the chart. It is not good practice to place figures at the upper ends of vertical bars, as the effect is to add to their lengths, thereby giving a false impression of the com-

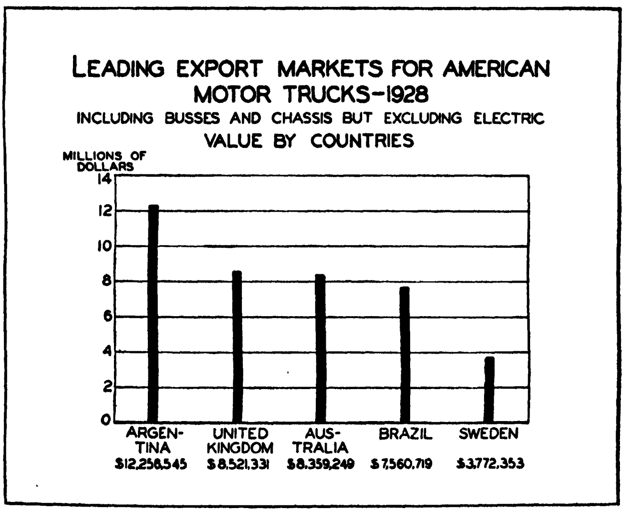

Exhibit 33.—Designating bars and tabulating data on vertical bar charts.

parison. When no scale is used and only a few bars are shown, the designations are sometimes placed within the bars, and for popular presentation this is quite effective but it has no advantage over the position illustrated in Exhibit 33.

Exhibit 33 also illustrates the point that, in designating bars, words should be divided if they are too long for the spaces allowed. They should also be lined up with reference to the zero line as shown.

Designations explaining what is represented by horizontal bars are placed on the left as illustrated by Exhibit 34*A* and *B*. The values represented also may be tabulated as shown in these

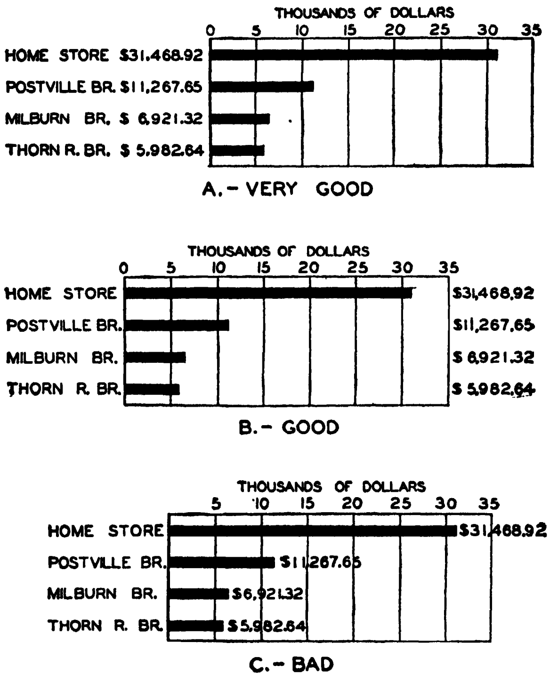

Exhibit 34.—Designating bars and tabulating data on horizontal bar charts.

illustrations (Exhibit 34*A* and *B*). The arrangement of Exhibit 34*A* is preferred, as it is easier to read than *B*, especially if there are great differences in the lengths of the bars. Numbers should not be placed at the ends of horizontal bars as in Exhibit 34*C*, because the effect is that the lengths of the numbers are added to the lengths of the bars and a wrong impression of the relative values is given. A still more objectionable plan is to block in designations and data within the bars as in Exhibit 35. This objection applies to both vertical and horizontal bars. In such cases it is probable that the reader will unconsciously compare

the solid parts at the right of the narrow column of figures instead of the entire lengths. In Exhibit 35 one cannot tell what is represented by the columns of figures, since no captions or other explanations are given. Charts always should be complete in such details, leaving nothing which might be unintelligible or falsely read to be explained in an accompanying text.

Another criticism of Exhibit 35 is that the lettering at the extreme left and the numbers of the scale read from top to bottom, while they should read from bottom to top if they are to be read vertically. It would be better to turn the scale

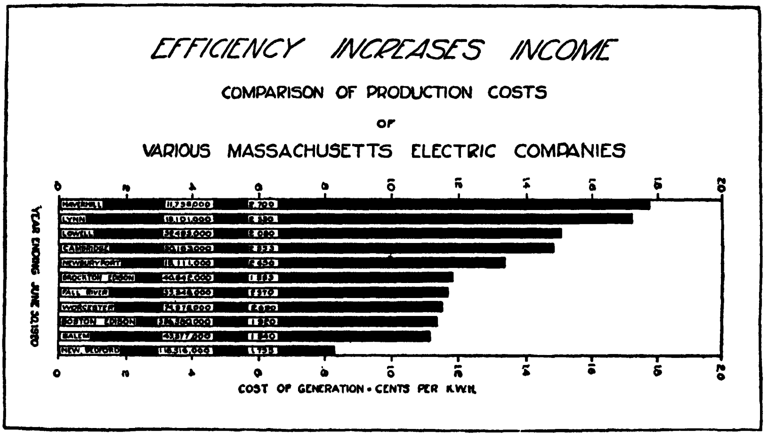

EXHIBIT 35.—Bad practice. Blocking-in has the effect of breaking the bars. (Also lettered incorrectly and lacks explanation of units.)

numbers so that they could be read horizontally. In this particular case (Exhibit 35) it is possible that the chart was made as a vertical bar chart and turned to a horizontal position by the party who later used it.

Common Errors.—Exhibit 36 illustrates various bad practices which are often followed by novices in graphic work. There is such a wide difference in the values that, in order to make oats show up well, wheat would be a very long bar. In such cases, the objective of showing the true relation is often overshadowed by the desire to show the small item. Very small numbers are practically insignificant when compared with large numbers. That is, $100, for instance, is practically negligible when compared with $1,000,000 and should be shown in that light. Exhibit 36 represents a hypothetical case, but all of the

shapes and methods illustrated were taken from charts published in reports and magazines.

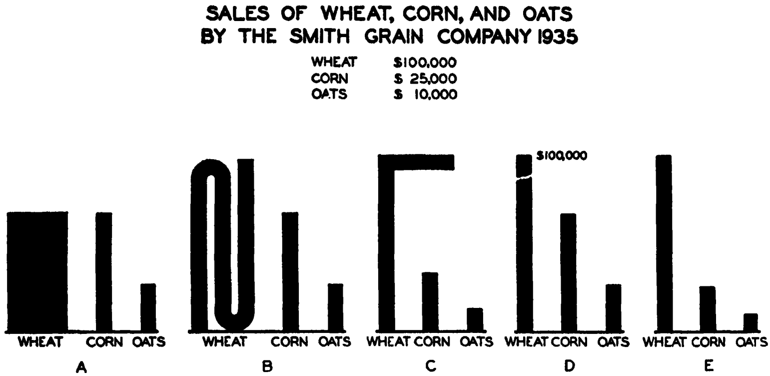

EXHIBIT 36.—Right and wrong practices. *A*, Wrong; the first bar is four times as wide as the others—lengths only should vary. *B*, Wrong; very hard to compare lengths. *C*, Wrong; difficult to compare lengths. *D*, Wrong; correct visual comparison is impossible because part of the bar is not shown. *E*, Right. (A scale and background of guide lines would further improve this chart.)

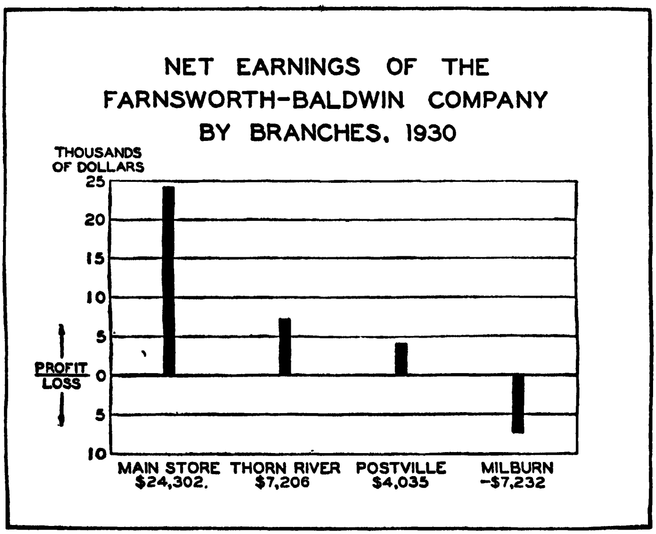

EXHIBIT 37.—Showing loss and gain with vertical bars.

Showing Decreases or Losses along with Increases or Gains.—So far, this discussion of bar charts has covered only positive numbers, which are represented by bars extending vertically

above the zero line or horizontally to the right of the zero line. When decreases or losses are represented along with increases or profits, they are commonly shown as in Exhibit 37 or as in Part *A* of Exhibit 38; that is, the negative values are represented by vertical bars extending downward from the zero line or by

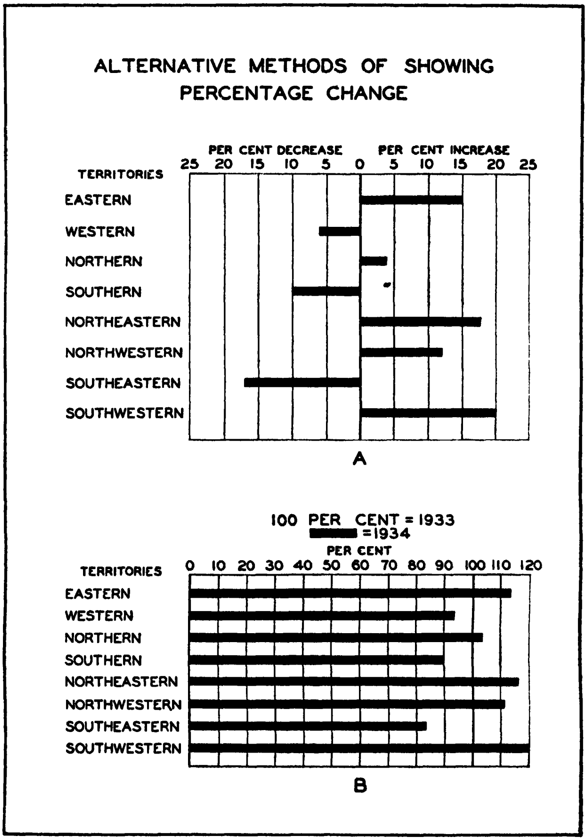

Exhibit 38.—Showing loss and gain with horizontal bars.

horizontal bars extending to the left from the zero line. Part *B* of Exhibit 38 illustrates another method of presenting the percentage change shown in Part *A*. In both parts of Exhibit 38, the problem is to compare the territories with reference to the changes that have taken place since the previous year. In *A*, simply the increases and decreases are represented. In *B*, the

previous year is considered as 100 per cent, and the percentage that the present year is of the previous year is represented. Both methods are clear and accurate, but the one used in *A* is the most easily understood by the average person.

Systematic Order of Items.—The items represented in any kind of presentation should always be arranged with reference to some systematic order. The most common in size comparisons is the order of size, but it is often preferable to use some other order, such as the alphabetical, geographical, or age. In

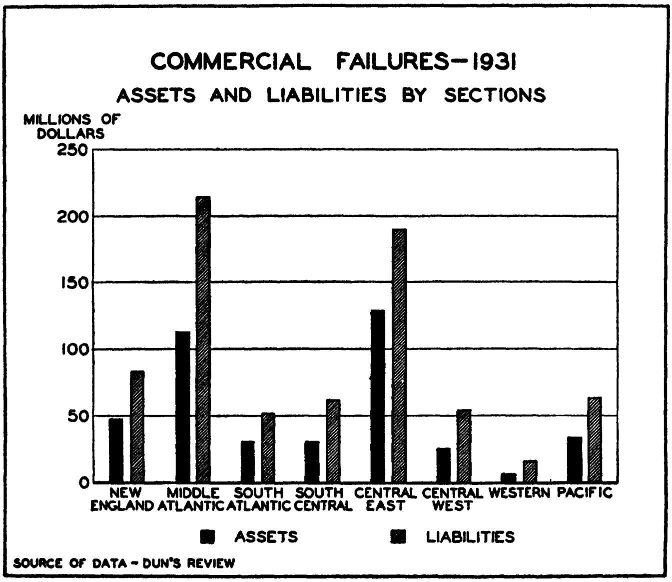

EXHIBIT 39.—Comparing different groups with vertical bars.

any case, some systematic order should be used, for nothing is lost and both clearness and ease of reading are added.

Comparisons of Differences.—Bar charts are often made, not only to show the differences between the sizes of various aggregates, but also to compare the differences between certain groups. Exhibit 39 on commercial failures makes a comparison of assets by sections, a comparison of liabilities by sections, a comparison of assets and liabilities in each section, and a comparison of the differences within each pair of assets and liabilities by sections for the group as a whole. Thus, one sees that failure liabilities were greatest in the Middle Atlantic and Central Eastern Sec-

tions. And one also sees that in the Middle Atlantic States liabilities were almost twice as great as assets, while in the Central East liabilities were much less than twice as great as assets. Furthermore, these different relations between assets and liabilities can be compared readily throughout the eight sections.

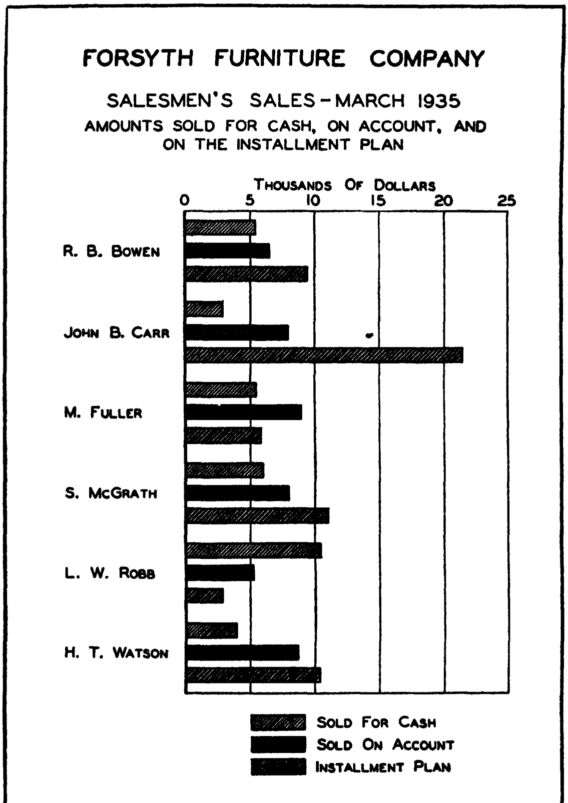

EXHIBIT 40.—Comparing different groups with horizontal bars.

In Exhibit 40 the differences in the types of sales made by the different salesmen may be compared. This comparison could be made with component bars (see Exhibits 48, 49, 50, and 59) which would bring out more clearly the totals sold but would not show so clearly the actual differences in sales by classes.

When comparisons such as those in Exhibits 39 and 40 are made, the bars should be arranged in pairs or groups as shown.

Thus, in Exhibit 39, there are two items (assets and liabilities) shown for each of eight geographic sections, and the bars are arranged in pairs with enough space between the pairs to separate them distinctly.[1] These pairs can then be compared easily and the differences considered. The chart in Exhibit 40 would be much more difficult to read if the bars were not grouped for each salesman. Whatever the arrangement may be in such charts, there should be substantially more space between pairs or groups than between the bars within each pair or group.

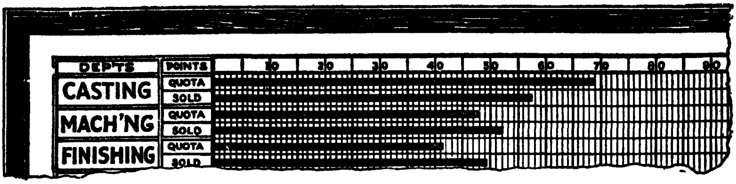

Exhibit 41.—Mechanical wall chart for comparing different groups. (*Courtesy of Educational Exhibition Co.*)

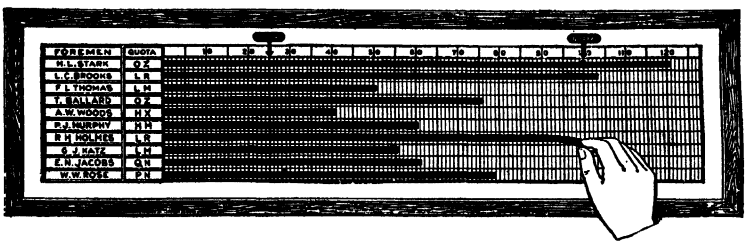

Exhibit 42.—Mechanical wall chart. (*Courtesy of Educational Exhibition Co.*)

In such cases as those illustrated in Exhibits 39, 40, and 41, it is necessary so to shade or color the bars that they will not be confused. The original chart of Exhibit 41 has a green ribbon for quota and a red one for work done. In Exhibit 39 the solid bar represents assets throughout the comparison, while the cross-hatched bar represents liabilities throughout, and there is one of each for every section. The explanations of the different kinds of shading or coloring may be made as in Exhibit 41, in which all dotted bars are designated "quota" and all solid

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[1] A simple method of laying out the irregular spacing of these bars to fit a given space is described on p. 224.

bars are designated "sold," or they may be explained as in Exhibits 39 and 40, in which a key or legend shows what each kind of shading represents throughout.

Common Unit Necessary.—It is necessary, of course, to use a common unit in making all comparisons of size. It means nothing to make a chart comparing, for instance, 10 dozen oranges and 50 pounds of prunes. The unit of measurement must be one that is common to both items compared.

Mechanical Bar Charts.—The charts shown in Exhibits 41, 42, and 43 are mechanical wall charts. The one shown in Exhibit 42 is made in several forms and combinations for showing

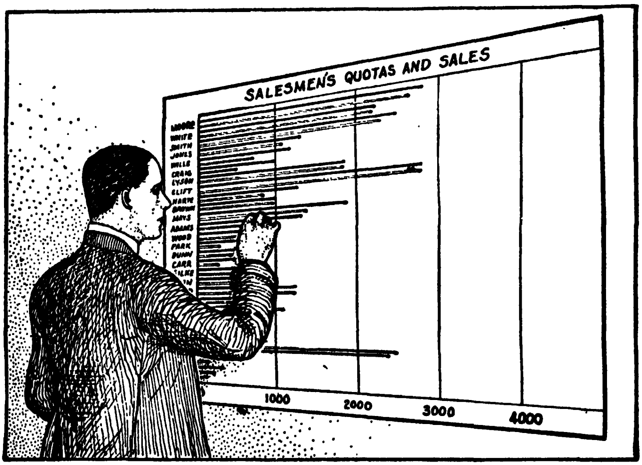

EXHIBIT 43.—A homemade mechanical wall chart.

sales, production, stocks, and other data. The bars are movable ribbons, which may be made longer or shorter at will by setting the tabs at the ends. The type of chart shown in Exhibit 41 is useful in making such comparisons as salesmen's or department's quotas with their actual sales, as the chart will show clearly how much the quota is under- or oversold. The background lines representing minor divisions are not necessary for reading the chart, but they are helpful in setting the movable bars.

Exhibit 43 shows a mechanical bar chart which can be made easily and cheaply. This particular chart shows actual sales and quotas of salesmen. Each salesman's name is placed on the left margin, and to the right of this are two small holes made by a pin. Through the top hole runs a red string (black

in the illustration) representing the quota, and through the bottom hole runs a blue string (white in the illustration) representing actual sales. The small holes hold the strings and they may be stretched taut across to the tacks at the movable ends of the bars. The board itself may be covered with cross-section paper.[1]

Comparisons of the Component Parts of an Aggregate.—Up to the present point this chapter has been devoted to comparisons of separate aggregates and comparisons of differences between groups of aggregates. Another important problem in size

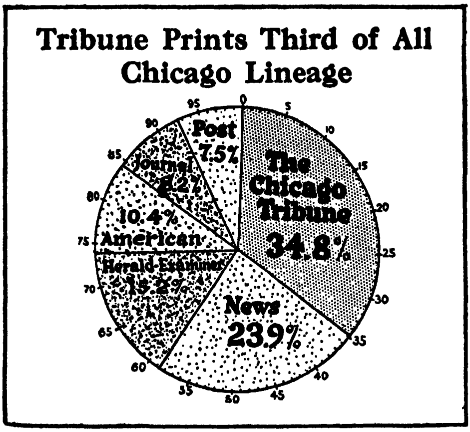

Exhibit 44.—A pie chart correctly made. (*Courtesy of Chicago Tribune.*)

comparisons is that of comparing the component parts of an aggregate, one with another or one with the whole. The method illustrated in Exhibit 44, commonly called the "pie chart," probably is the most generally known because of its wide use in popular presentations. This form has been criticized on the ground that it is difficult to compare the relative sizes of the sectors, but this is not a serious fault since practically everyone begins early in his career to practice judging the relative sizes of sectors of pies and is quite familiar with the process of judging such shapes. It is probable, however, that many persons can read a component bar chart (Exhibits 49 and 50) more accurately than a pie chart.

[1] Another mechanical bar chart (presenting a time series) is illustrated on p. 59.

The Pie Chart.—The pie chart permits a convenient arrangement of the designations of the different components as well as the percentages and actual values represented. The wording should be placed inside the sectors when possible, and it always should be so placed that it reads horizontally (Exhibit 44). The wording never should be placed in such a way that it reads radiating from the center or following the circle as in Exhibit 45, for such arrangements of lettering are much more difficult and confusing to read than horizontal lettering. Exhibit 45 attempts to

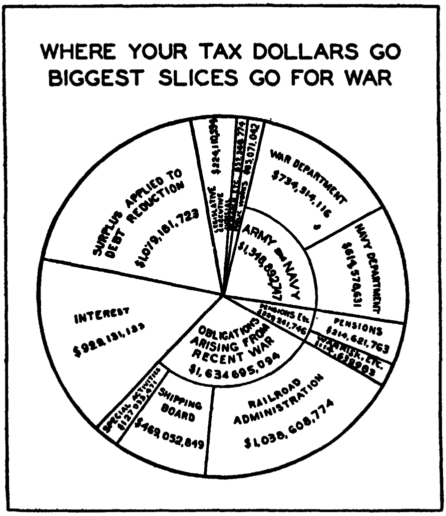

EXHIBIT 45.—A pie chart not correctly made.

show a wide range of extremes and minor subdivisions within the major sectors. Usually these small items are not important enough to be shown separately, but, if this is desirable, a better method of dividing a sector, or showing components of a component, is to use light or even dotted lines to mark the subdivisions, while the major sectors are limited by heavy lines. Also if a sector must be subdivided, it should be cut by a line radiating from the center, and not by an arc drawn across the sector.

If a pie chart is to be most effective, the sectors should usually be made to stand out separately by coloring or shading. Care should be taken so to choose and arrange colors or shading that they will not emphasize the importance of some sectors as compared with others, but will show only the fact that different items

are represented. For instance, a solid black sector will stand out more prominently than a cross-hatched sector of the same size, or a bright red will stand out more than a dull gray. The size alone of the sector should signify its importance.

The sectors of a pie chart should be arranged according to some systematic order. Usually they are arranged in order of size, and the most generally followed order is from large to small in a clockwise direction beginning at the top as illustrated in Exhibit 44. Of course, a sector representing a combination of several

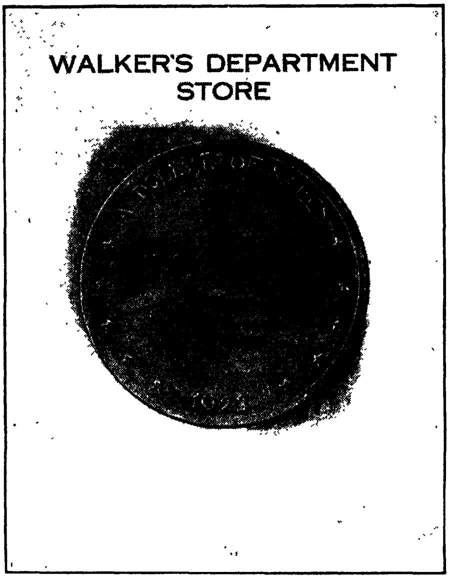

Exhibit 46.—A pictorial pie chart.

small items, and designated "all others," naturally would be given last place even if it were larger than some of the other sectors.

In making popular presentations, pictures are sometimes placed on the sectors. When using this method, one should not make the illustrations too prominent, because the eye may compare the sizes of the illustrations rather than the sizes of the sectors. This type of practice should be followed only in the most simple forms of charting, such as those used in certain popular magazines and newspapers or in children's books and magazines.

In making an appeal to certain types of business men, a very effective application of the pie-chart principle is shown in Exhibit 46. Exhibit 47 shows another presentation of similar figures which is easily understood by the average reader. It is simply a bar chart made up in popular form to represent the component parts of a dollar. The ordinary bar chart is often used in this manner to show component parts.[1]

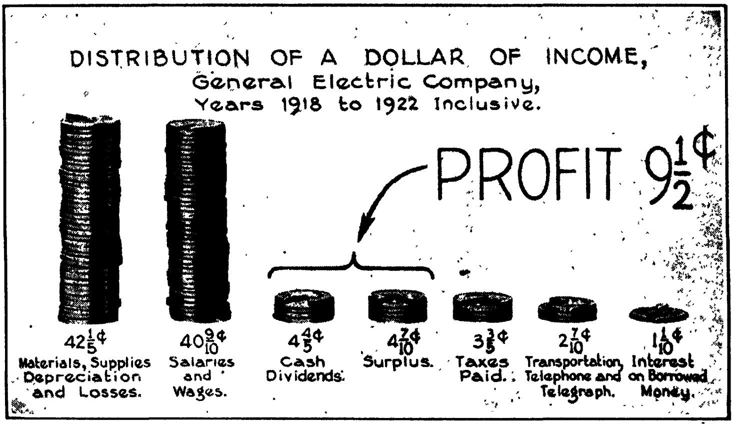

EXHIBIT 47.—Another pictorial method of showing data similar to that of Exhibit 46. (*Courtesy of General Electric Co.*)

The Component Bar Chart.—An excellent form for showing the component parts of a whole is the component bar (Exhibits 48, 49, and 50). The component bar may extend either vertically or horizontally, but the direction should be chosen that will make the chart most easy to read. The designations of the different components may be placed within the bar (Exhibits 48 and 49) or outside the bar as in Exhibit 50. It is well to show scales on these charts as a further aid in reading and checking (see Exhibits 48 and 50). If designations are placed outside a vertical bar, it is usually best to place them at the right of the middle of each

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[1] For additional discussions of pie charts and component bars, see Walter Crosby Eels, "The Relative Merits of Circles and Bars for Presenting Component Parts," *Journal of the American Statistical Association*, June, 1926: R. von Huhn, "A Discussion of the Eels Experiment," and Frederick E. Croxton, "Some Additional Data," *Journal of the American Statistical Association*, March, 1927.

component as in Exhibit 50. This leaves the left side free for a scale, while designations of uneven length will be close to their components and still begin in a straight vertical column.

The components should be arranged in a systematic order such as size. If the order of size is followed, a vertical bar will appear

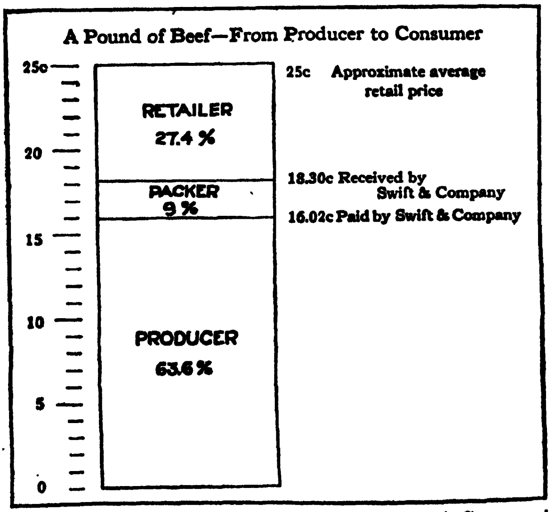

EXHIBIT 48.—A vertical component bar. (*Courtesy of Commercial Research Department, Swift and Co.*)

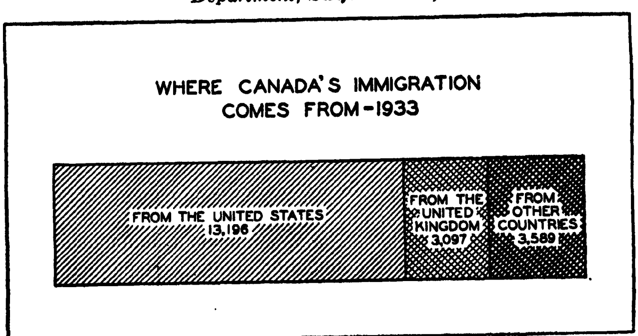

EXHIBIT 49.—A horizontal component bar.

less topheavy if the larger sections are at the bottom. In Exhibit 48 the order of size was not used since the packer naturally comes between the producer and the retailer.

The discussion of differentiating pie-chart sectors by coloring and shading applies equally well to bar components. While it

is well to differentiate the components, care must be taken not to emphasize some of them by using too strong cross-hatching or too brilliant colors.

Complicated Forms Should Be Avoided.—Exhibit 51 is a combination of a component bar and simple bar chart. The number of pounds of the different cuts of beef are shown in the horizontal component bar. The vertical length of each component represents the price of the cut. The area of each rectangle represents the total amount received for the cut. This chart, however, is not easy for the average reader to understand, and similar figures for the following year were represented as in Exhibit 52 which tells the story very simply and clearly. An objection to the plan used in Exhibit 52 is that the bars representing pounds extend to the left of the base line. This practice should be reserved for showing decreases, losses, and other negative quantities, while horizontal bars representing positive figures should extend to the right of the zero base (see Part *A* of Exhibit 38 on page 35).

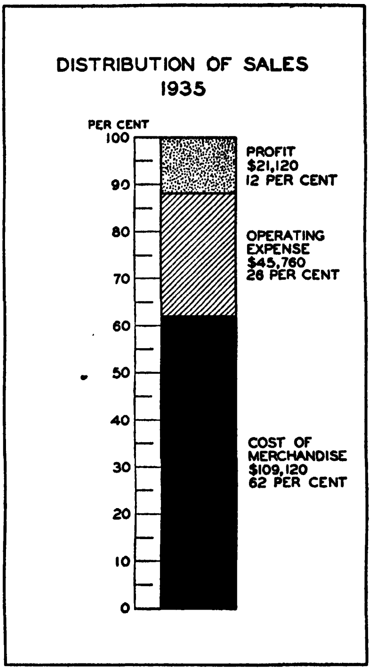

EXHIBIT 50.—A vertical component bar with scale.

In the sales profit and loss analysis chart shown in Exhibit 53, the aim is to give a picture of the profits made on the different types of merchandise. *A*, *B*, *C*, *D*, and *E* are symbols representing the various classes of commodities. The vertical scale shows the percentage of profit or loss on each commodity. The horizontal scale shows the volume of sales of each commodity expressed as percentages of the total. The area of each rectangle represents the amount of profit or loss on each commodity. An important objection, however, to this chart is that it is difficult to compare the different areas. Another method of making the

same comparison is shown in Exhibit 54. This form is more simple in construction than Exhibit 53 but it has three times as

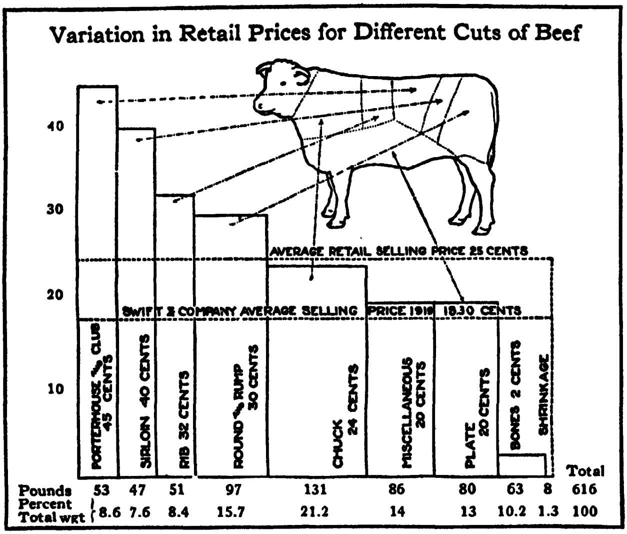

EXHIBIT 51.—A complicated form. See Exhibit 52. (*Courtesy of Commercial Research Department, Swift and Co.*)

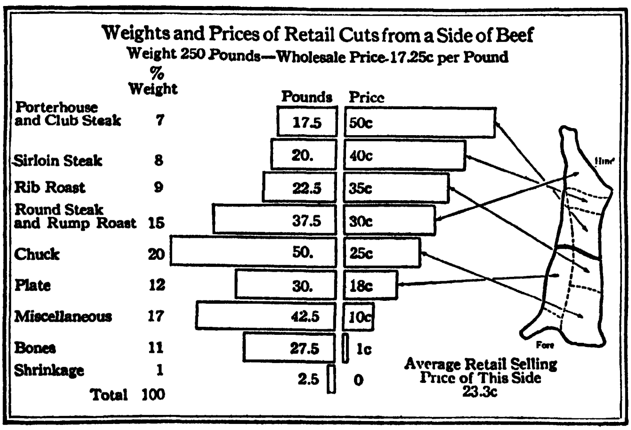

EXHIBIT 52.—A better form for showing similar data to that of Exhibit 51. Note, however, that the practice of extending bars to the left of the zero base should ordinarily be reserved for showing minus quantities. See Exhibit 38, Part A. (*Courtesy of Commercial Research Department, Swift and Co.*)

many bars. As a general rule, however, it is well to keep away from the use of forms that involve the comparison of areas if it is

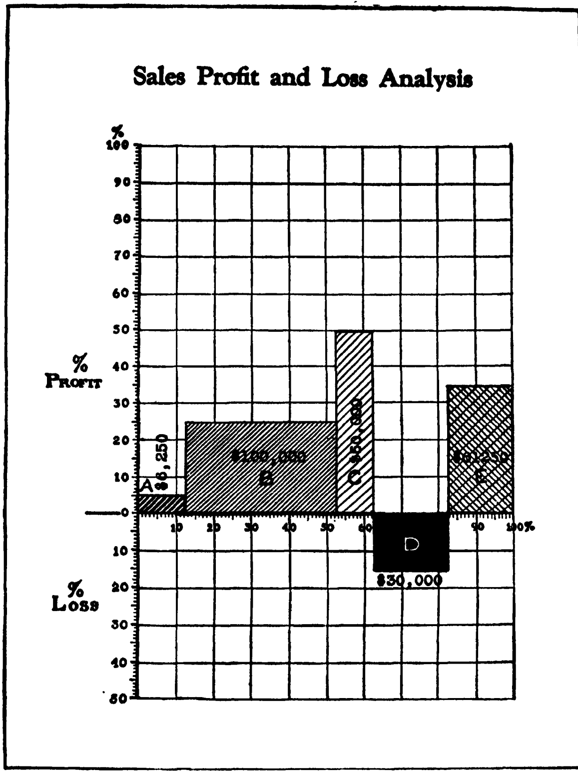

EXHIBIT 53.—Complicated forms like this should be avoided.

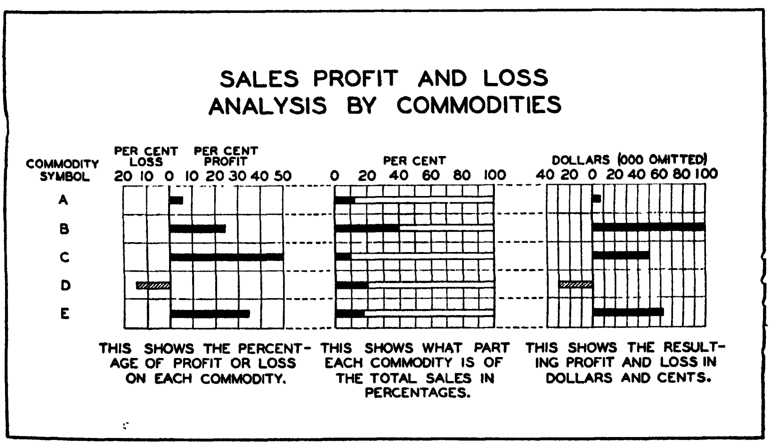

EXHIBIT 54.—A more simple method of showing the data of Exhibit 53.

possible to use forms that vary in only one dimension. It should be noted that the percentage of the total is expressed in the

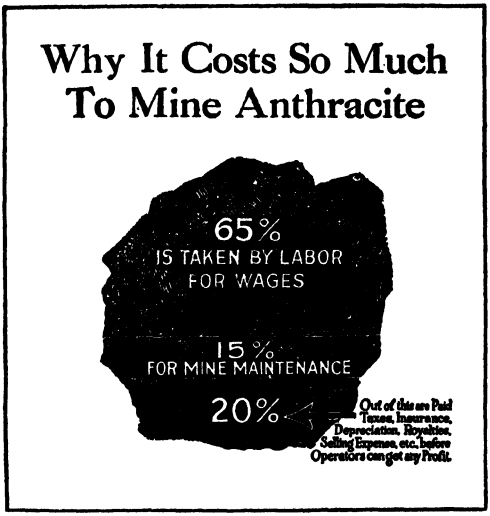

Exhibit 55.—A pictorial component bar. (*Courtesy of Anthracite Operators' Conference.*)

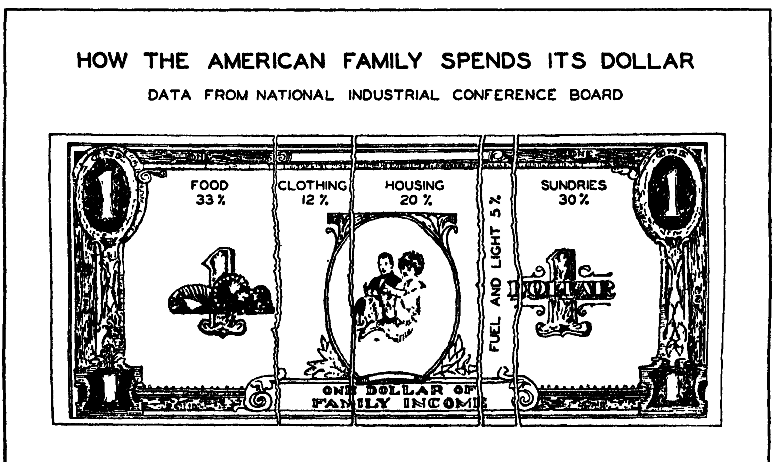

Exhibit 56.—A pictorial component bar.

central group in Exhibit 54 by indicating 100 per cent in outline and showing the percentage for each commodity in solid black.

The loss figures are shown in red on the originals of both Exhibits 53 and 54.

Pictorial Forms for Showing Component Parts.—In Exhibits 55, 56, and 57 the aim is to add interest and clearness by picturing what is represented as well as to show the various component parts. The irregular shape of the lump of coal in Exhibit 55 does not permit easy or accurate measurement by the reader, but it is quite effective in showing the divisions roughly and this is the purpose of the chart. For popular reading Exhibit 56 is a

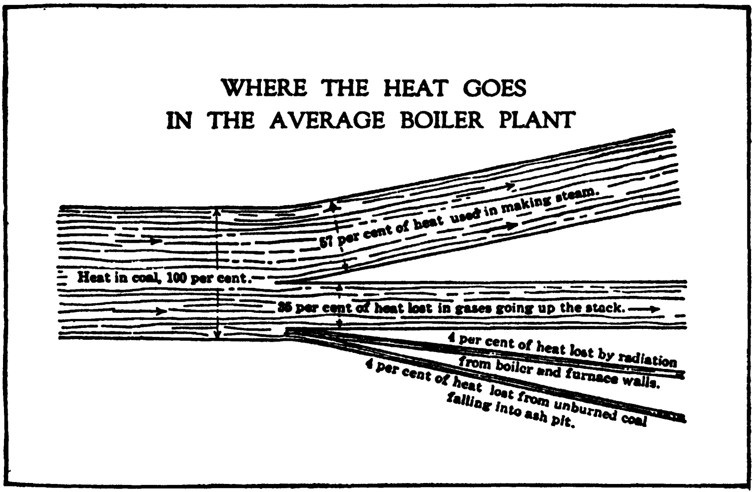

EXHIBIT 57.—A pictorial component bar. (*Courtesy of National Engineer.*)

good presentation to show roughly the distribution of living expenses. Exhibit 57 is an effective presentation of the relation of heat used in making steam to the rest of the heat in bituminous coal.

Component Parts of Component Parts.—A method of showing component parts of component parts is shown in Exhibit 58. If a component must be subdivided, this method is effective only for presentation to a trained reader. Since this problem is not a common one, the form shown in Exhibit 59 is better for general use. Here the major components are separate bars which show the subdivisions representing minor components within each bar.

Comparisons of Aggregates Made Up of Components.—Up to this point, the consideration of component parts has included

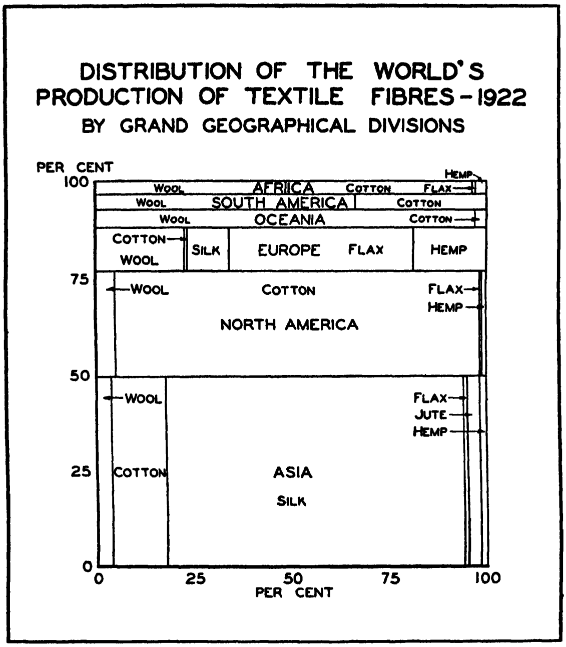

EXHIBIT 58.—A poor method of showing component parts of component parts. (Compare with Exhibit 59.)

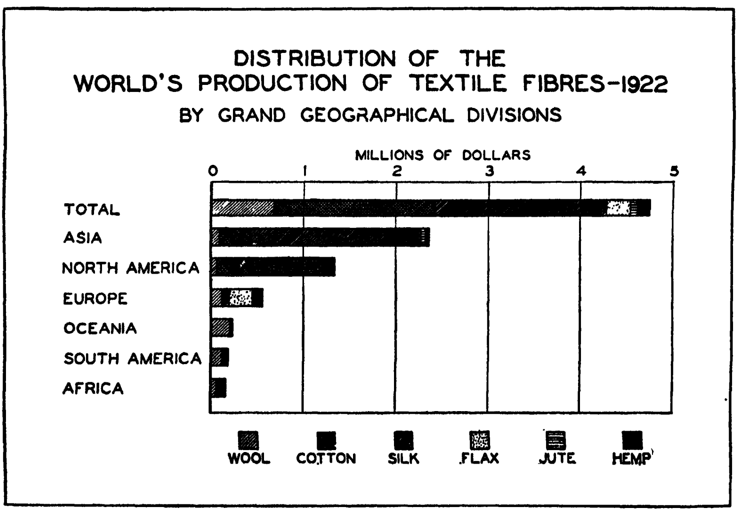

EXHIBIT 59.—Usually a better method of showing the data of Exhibit 58.

only the comparison of those parts which are within single aggregates. Often it is desirable to compare aggregates made up of component parts, either with reference to the actual sizes of the

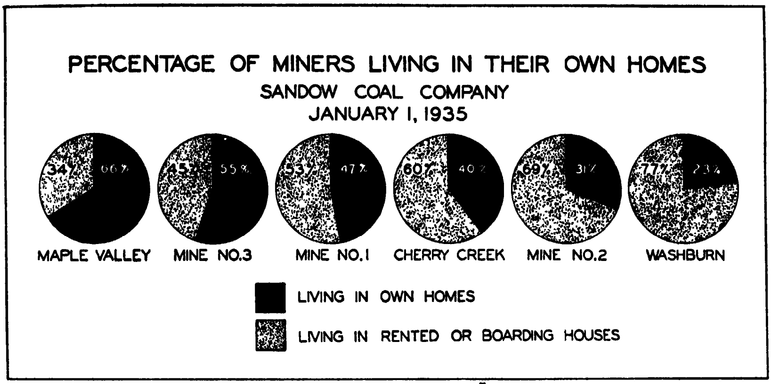

EXHIBIT 60.—Use of pie charts in comparing the percentage distributions of the component parts of different aggregates.

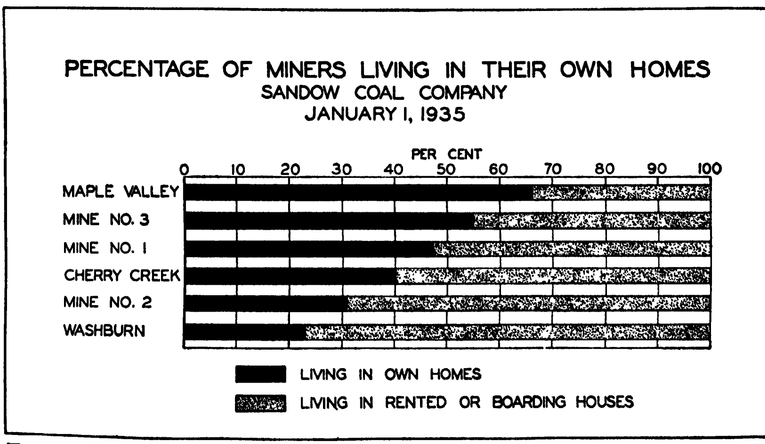

EXHIBIT 61.—Use of the bar chart in comparing the percentage distributions of the component parts of different aggregates.

aggregates, or with reference to the percentage distribution of the parts within the aggregates. If the problem is one of comparing the percentage relations of the parts to the whole, either the pie chart or the component bar chart can be used, as the dif-

ferent forms representing 100 per cent would be the same size (Exhibits 60 and 61). When the bar form is used in careful work, it is best to show a scale and a background of guide lines as in Exhibit 61. If, however, the problem is one of comparing actual sizes of aggregates and their components, the bar method should be used in preference to the pie chart. Circular areas (Exhibit

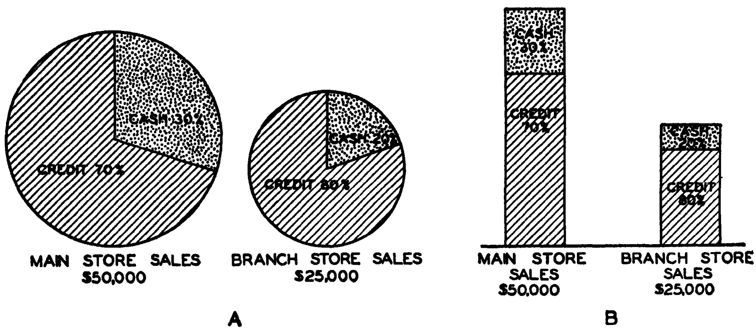

EXHIBIT 62.—Comparison of aggregates which show component parts. *A*, Poor practice, as sizes of circles are difficult to compare. *B*, Good method, as it is easy to compare bars of different lengths.

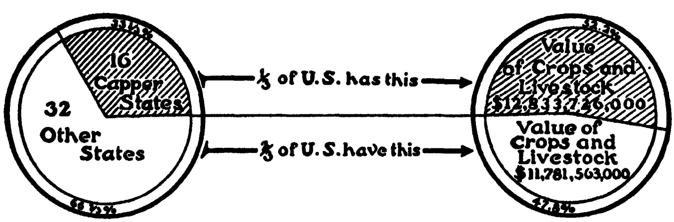

EXHIBIT 63.—Comparison of percentage distributions for popular presentation. (*Courtesy of Capper Farm Press, Copyright*, 1920.)

62*A*) are hard to compare, but this difficulty does not arise in using component bars (Exhibit 62*B*).

In both percentage and actual value comparisons, the components should be differentiated by shading or coloring—each shade or color representing the same thing in all the bars or circles. When only a few aggregates are compared, the designations may be placed on the components, as in Exhibit 62; but when many items are compared, it is easier to read the chart if a key or legend

descriptive of the cross-hatching or coloring is placed on it as in Exhibit 61.

Exhibit 63 illustrates a method of comparing distributions that is very easy to understand. The relation of one pie chart to the other is clearly indicated by the descriptive matter and arrows between the charts.

A Common Error in Using Component Bars.—The purpose of component bar charts and the nature of the data represented

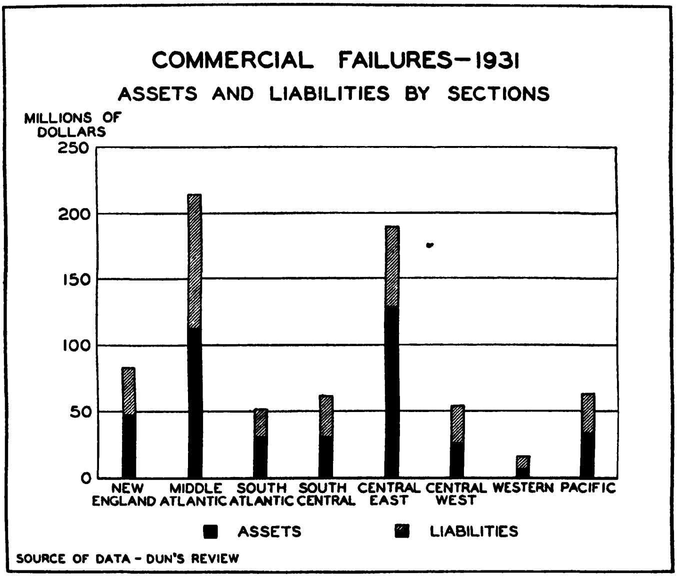

EXHIBIT 64.—A common error. Incorrect, as the chart does not show component parts. (See Exhibit 39 on page 36.)

must be clearly understood and should not be confused, as is the case in Exhibit 64 where the black portion and the cross-hatched portion of each bar are not the component parts of an aggregate. In this figure, both an amount represented by a solid black bar and an amount represented by a cross-hatched bar are measured from the zero line, and it is not easy to imagine the cross-hatched bar to extend from the zero line with the solid black bar superimposed upon it. Assets and liabilities are separate aggregates and should have been shown by separate bars as in Exhibit 39 on page 36.

Checking List.—The following list includes the most important points to be observed in making the charts described in this chapter, and can be used when checking a chart to see if it is complete:[1]

Did you check for accuracy?

Is the title simple and clear? Did you place it at the top of the chart?

If you have used shadings or colors to make any necessary distinctions, have you included a legend or key to the meanings of the shadings or colors?

Have you placed a convenient scale and a background of lines upon the chart to aid in measuring sizes?

Are the faces of the bars free from these lines?

Is the zero line heavy?

Are the other scale lines light?

Have you designated the scale units?

Have you given the reference or source?

Does all lettering read horizontally? (When impossible to letter horizontally, lettering should read from bottom to top—never from top to bottom.)

Are your letters formed according to a commonly accepted style of plain lettering?

Have you ruled in a border line? (Optional but advisable.)

If desirable, have you included your name (lower right) and date (lower left)?

Are pencil lines erased?

Will your chart be clear to the reader?

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[1] A checking list for curve charts is presented on p. 138.

CHAPTER IV

HISTORICAL TIME SERIES—ARITHMETIC CHARTS

In Chapters IV and V the purpose is to describe the graphic methods of presenting data with reference to definite *dates*. The comparison of phenomena at different *dates* probably is the most important field in business statistics. Thus we may wish to compare our sales for last month with our sales for every other month for the past five years in order to determine whether or not we are gaining or losing trade. By presenting these figures graphically, we are able to see at a glance a picture of the whole period.

This study of presenting time series is divided into five distinct parts as follows:

1. Size changes in a single variable over a period of time.
2. Comparisons of size changes in different time variables.
3. Components of a time variable.
4. Rates of change in a single variable.
5. Comparisons of rates of change in two or more variables.

Number 1 above would be illustrated by a chart of a single series showing, say, amounts of sales by months. Then the part of the curve for the latest month could be compared with the parts of the curve showing amounts of sales in the same business for other months. Number 2 would be illustrated by a chart of more than one series comparing the volume of sales of the main store with the volumes of sales of, say, its branches for the same period. Number 3 might show over a period of time our sales of, say, shoes and how these sales were divided into men's, women's, and children's, or how the total was divided into parts sold by each salesman. Number 4 might show, for instance, whether our sales were changing at an increasing or decreasing *rate* as compared with the past. In Number 5 one might compare sales of the main store with those of its branches, not to show how much larger the main store is than the others, but to show by the slopes of the curves which store is increasing its sales at the most rapid rate.

The five parts listed on the preceding page are distinct forms and should be recognized as such. One cannot be used safely unless the others are understood, and one cannot take the place of another. They will be considered in the order listed.

SIZE CHANGES IN A SINGLE VARIABLE OVER A PERIOD OF TIME

In this kind of comparison, the problem is to show the sizes of the phenomena at different dates. Thus, one might compare

Exhibit 65.—Pictorial chart of time series. (Unequal time intervals are sometimes permissible in pictorial charts.) (*Courtesy of Automobile Manufacturers Association.*)

the amounts of labor needed to buy a car at different dates as shown in Exhibit 65. The study is confined to a single series.

Simple Pictorial Time Charts.—When the problem is one of popular presentation, the pictorial chart is very effective, but, in all cases, a geometric form should be chosen which can be made to vary in a single direction without distortion. Thus, in Exhibit 65, the comparison is clearly made since the piles of lumber vary only in their heights. In Exhibit 66 the quantities of coal mined in the United States in 1926 and 1932 are compared. The amounts represented are proportional to the heights of the piles. As the reader unconsciously compares the volumes,

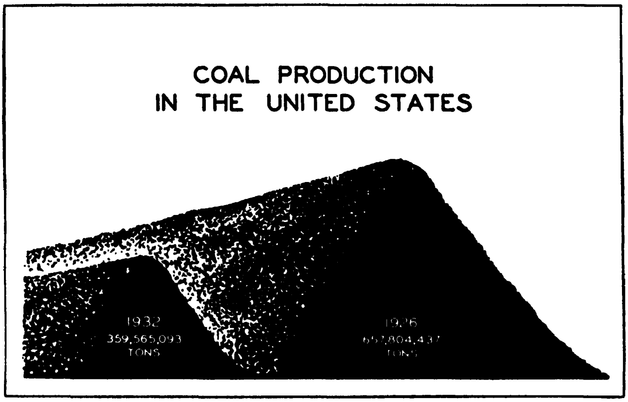


EXHIBIT 66.—Pictorial comparison of time data. (Wrong because volumes are misleading. Also chronological order is reversed.)

he gains a wrong impression of the sizes represented. The large pile represents twice as many tons as the small one, yet it contains much more than twice as much coal. Volumes like those in Exhibit 66, varying in three dimensions, should not be used in graphic presentation, as they are likely to be misinterpreted. Exhibit 67, representing the amounts of money spent for candy during a 5-year period, is an effective chart for popular presentation.

Ordinarily the time scale should be laid out along the horizontal axis of a time chart, but occasionally it may be laid out vertically in popular presentations as illustrated in Exhibit 68. This rule, however, should not be violated in any but popular pictorial charts, and then only when distinctly necessary.

In planning a chart, it should be remembered that for the trained reader the pictorial form may add nothing. It is,

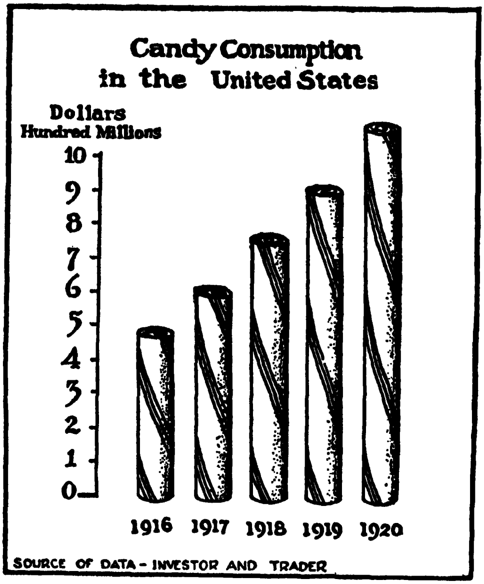


EXHIBIT 67.—Pictorial chart of time series for popular presentation. (Correctly drawn.) (*Adapted from Investor and Trader.*)

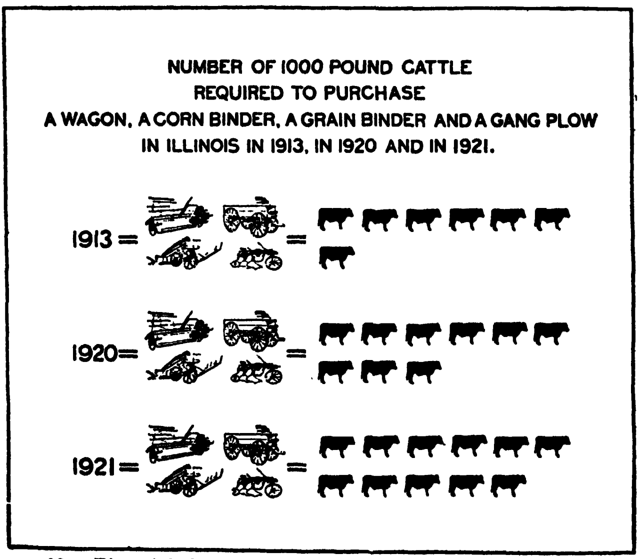


EXHIBIT 68.—Pictorial chart of time series for popular presentation. Pictorial charts are sometimes exceptions to the vertical scale rule stated in connection with Exhibits 70 and 71. (*Courtesy of United States Department of Agriculture.*)

however, an excellent method to use in popular publications and in children's books and magazines.

Exhibit 69 pictures a mechanical bar chart which is designed to attract attention through the use of the thermometer idea. The appearance of rising and falling liquids is accomplished through the use of power-driven ribbon belts. The thermometers rise to the proper heights in chronological order and then drop to zero. This process is repeated at intervals of about one minute. Variations of this method have possibilities of wide application in popular displays of statistical facts.

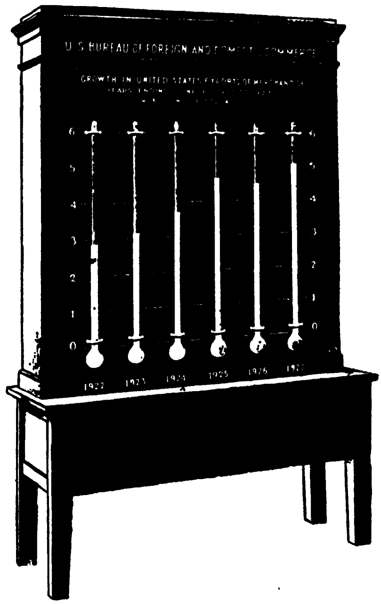


EXHIBIT 69.—Mechanical bar chart for popular display of statistical facts. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

Bar Charts for Time Series. Bar charts, similar to those described in the preceding chapter, are often used to show time series (Exhibits 70 and 71). In showing time series, however, it is much better practice to use the vertical bar chart (Exhibit 70) than the horizontal bar chart (Exhibit 71).[1] The reason for this is that the chart of vertical bars is read in exactly the same way as a time curve, which is by far the most common way of presenting these data. When the vertical bar chart is used for showing a series of positive figures over a period, the time scale should read from left to right as shown in Exhibit 70. This scale should be laid out in such a way that equal spaces represent equal time intervals over the period. That is, for instance, each inch might equal 1 year. This rule should be followed even though the data are not complete, as in Exhibit 72. Had the bars been evenly

[1] This rule does not apply, however, in making the simple comparisons of size described in Chapter III (see pp. 25–27).

spaced on this chart, a wrong impression might have been gained by the reader.[1] It should be noted, however, that this rule

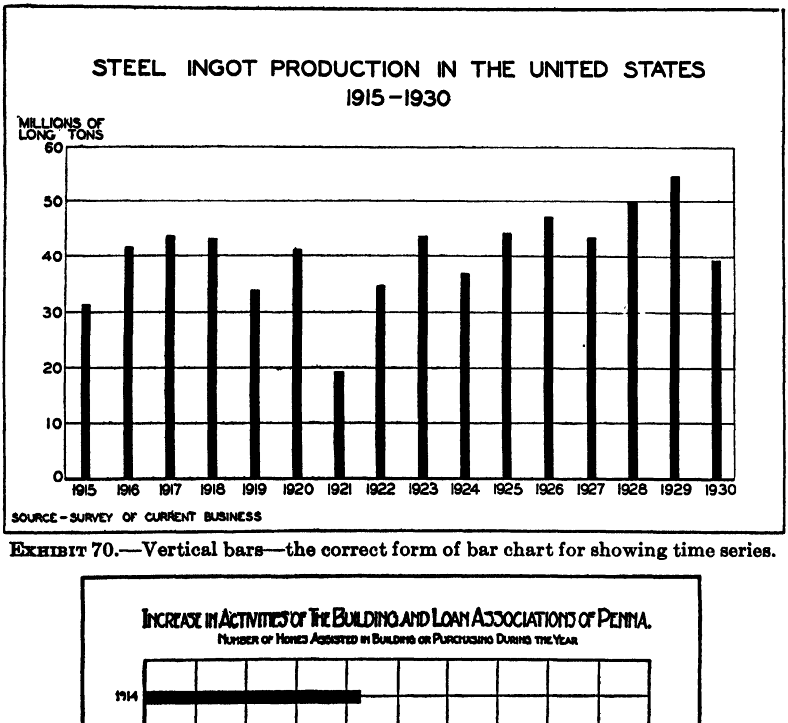


EXHIBIT 70.—Vertical bars—the correct form of bar chart for showing time series.

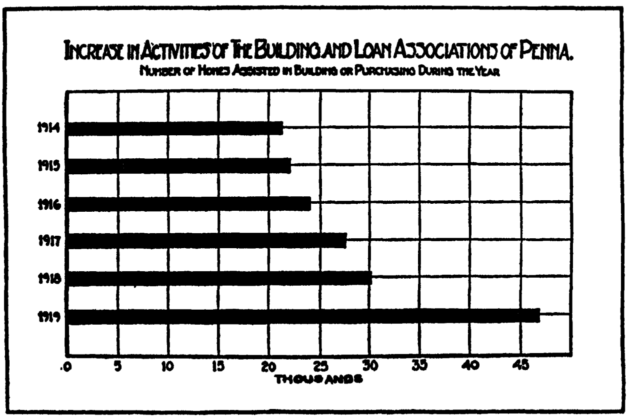


EXHIBIT 71.—Horizontal bars—incorrect for showing time series. (Abbreviations should not be used in titles.)

may sometimes be violated in pictorial charts that are used in popular presentations where the time variations are only of general significance (see Exhibits 65 and 68).

[1] See examples of errors in plotting curves of similar series in Exhibits 79 and 80 (pp. 67 and 69).

Curves of Size Changes.—The most common method of showing time series graphically is by means of a curve.[1] In Exhibit 73 the arithmetic time curve tells the same story as the bar chart and the similarity is illustrated.[2] A curve represents the ends

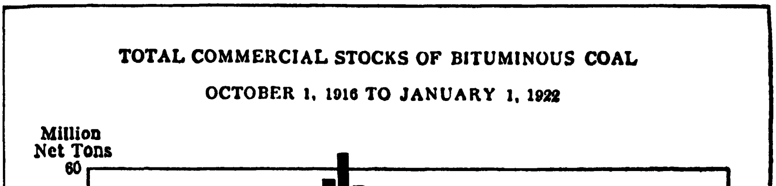
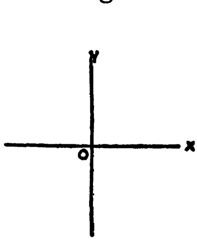


Exhibit 72.—Correct use of bar chart for showing incomplete series. (*Courtesy of United States Geological Survey.*)

of the bars, but by being continuous it indicates the direction of the change more clearly than would a bar chart.



[1] A very essential element in graphic presentation of statistics is the systematic use of *coordinates*, that is, numbers which determine the position of a point in a plane. *Coordinate axes* are two indefinite straight lines at right angles to each other. Their point of intersection, *O*, is called the *origin*. The line *OX* is called the axis of *X*, or the axis of *abscissas*. The line *OY* is called the axis of *Y*, or the axis of *ordinates*.

[2] This type of curve is referred to as an arithmetic time curve to distinguish it from other types of curves (logarithmic time curve, frequency curve, and correlation curve) which will be discussed later.

The general arrangement of a time curve should be such that it reads from left to right. Thus, as in Part *C* of Exhibit 73, we follow the curve over the period in the direction in which we naturally read. As far as possible, the lettering should be so placed that it can be read horizontally from left to right. When it cannot be so placed, it should read from bottom to top.

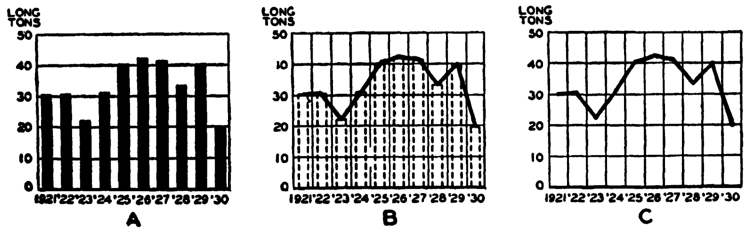


EXHIBIT 73.—Comparison of bar and curve methods of showing time series.

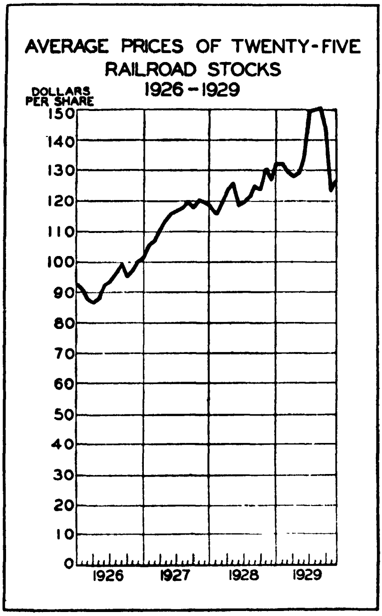
The Arithmetic Vertical Scale.—In the vertical scale for an arithmetic time curve, equal spaces represent equal values. Thus, in the original of Exhibit 74, one-half of an inch represented $10 anywhere in the scale. It should be noted, however, that expressions such as "½ inch = $10" are not placed on statistical charts. All necessary measurements can be read much more easily from the scale on the graph, and in photographic enlargements or reductions the scales are taken care of automatically.

The correct position of the vertical scale is on the left of the first ordinate (Exhibit 74). If it will add anything to the ease of reading the chart, a scale may be placed on the right of the last ordinate also, but in no case should the scale be omitted on the left. The figures of the scale should be exactly opposite the lines which they represent (Exhibit 74). The units in which the scale is laid out should be explained by a caption at the top of the scale.[1]

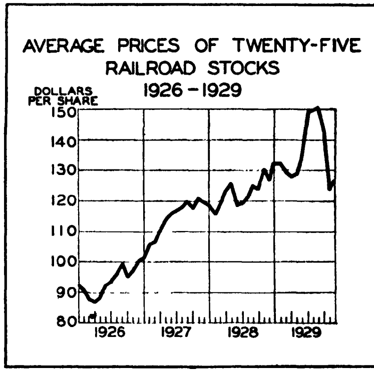
Importance of Including Zero on the Vertical Scale.—When the problem is one of comparing sizes over a period of time, the zero line and all of the scale must be included on the graph. If the lower part of the scale is not shown, as in Exhibit 74*B*, the reader is given either no definite graphic impression of the size changes—or a very inaccurate one. Such a chart is likely to

[1] See the discussion of captions or scale unit designations in the preceding chapter in connection with Exhibit 29 (p. 29).

give the impression that the prices represented have increased much more than they actually have, for only by referring to the scale can it be seen that the prices at the highest point have increased by only about two-thirds. The correct impression is given only when the whole scale is included as in Exhibit 74*A*, where the increase appears in its true relation to the earlier



A



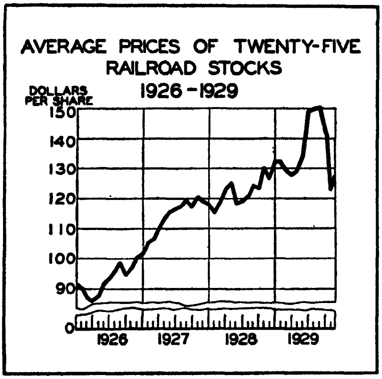
B

EXHIBIT 74.—Arithmetic time curve. *A*, Correct scale for making size comparisons. *B*, Incorrect for size comparisons because lower part of vertical scale is cut off. (See Exhibit 75 for exceptions.)

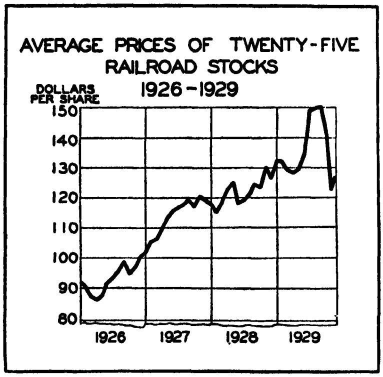
data. If it is desired to use only the space covered by Exhibit 74*B*, the vertical scale of the chart in Exhibit 74*A* may be contracted to fit, but the base should not be cut off in size comparisons. (This principle was discussed also in Chapter III in connection with Exhibits 26 and 27 on pages 27 and 28.)

Sometimes it is desired to show time changes without considering the size or rate of change. For instance, suppose that in Exhibit 74*A* the purpose is to show the fact that prices have started upward, without considering either the amount or the rate of the change that has taken place. In such cases, if the entire scale does not appear, attention should be called to the fact

by showing the zero line below a horizontal break in the graph, as in Exhibit 75*A*, or simply by showing a break at the bottom of the graph as in Exhibit 75*B*. Either of these warnings should prevent the reader from attempting a strictly graphic comparison of sizes for the different dates.



A



B

EXHIBIT 75.—If the entire scale is not shown, this fact should be emphasized by a break at the bottom of the chart, as in *A* or *B* above.

Exhibit 76 is plotted in percentage relatives,[1] with the 100 per cent line as the base above or below which the curve is plotted. The zero line of a percentage relative scale may be omitted if it is relatively a long distance from the 100 per cent line. If, however, the zero line is included, it is much easier to judge the values plotted. For instance, if the whole scale is given, and the curve drops to 75 on the scale, as in Exhibit 76, it is easier to appreciate that the curve has dropped one-fourth of the way to zero.

The Horizontal Scale.—The most commonly accepted position for the horizontal scale is along the bottom of the graph, as shown in Exhibits 76, 77, and 82. This position is the most convenient one for making comparisons from the zero base. If it will make the graph easier to read, a scale may be placed along the top in addition to the one along the bottom. However, the scale along the bottom should never be omitted. It is not necessary to

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[1] Percentage relatives of values at given times are the percentages that these values are of a value for a certain period taken as a base of 100. See further explanation on pp. 125 to 131.

designate or explain in what units the time scales are expressed, except when hours, days, or other units are used which are not

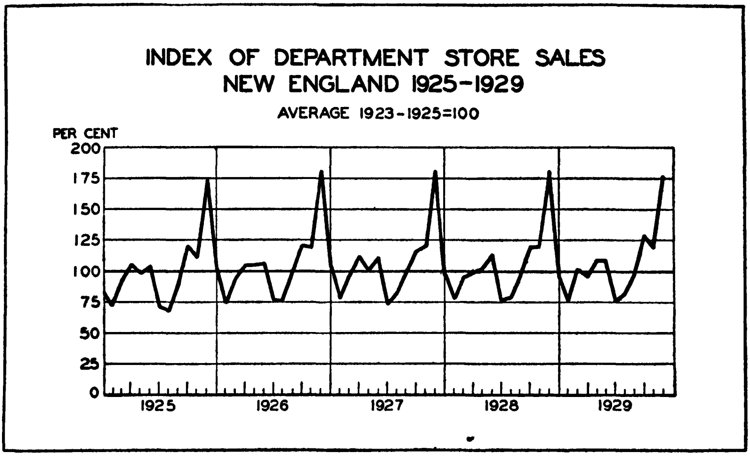

EXHIBIT 76.—Curve of percentage relatives. The zero line and the 100 per cent line should be emphasized. The curves, however, should be the heaviest ruling on the chart.

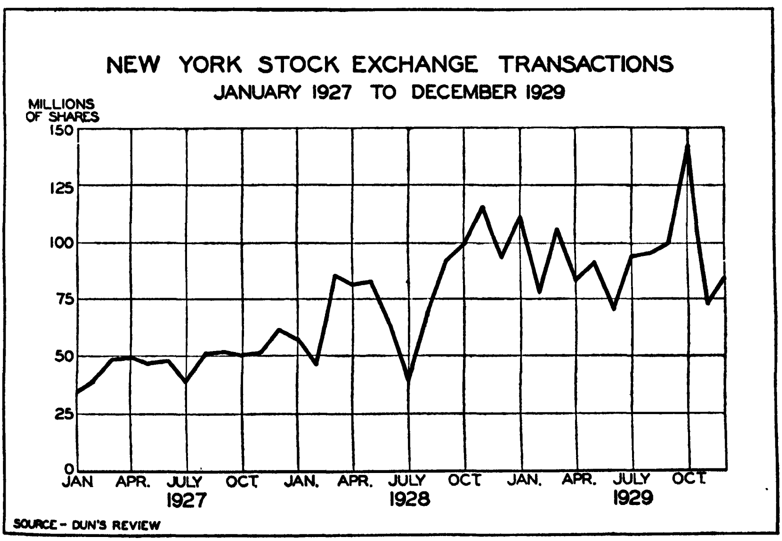

EXHIBIT 77.—Designating months by quarters.

clearly recognizable. Years or months are always clear without an explanation or designation. In making charts for general use,

it is better to designate months by their common abbreviations rather than by their first letters. Letters are confusing because the names of more than one month begin with the same letter. (January, June, and July begin with J, March and May with M, and April and August with A.) If there is not room to letter all the abbreviations horizontally, it is often desirable to letter only the quarterly months, as in Exhibit 77, in which the data are plotted monthly and the ruled lines and designations are on a quarterly basis. In designating by quarters, some statisticians

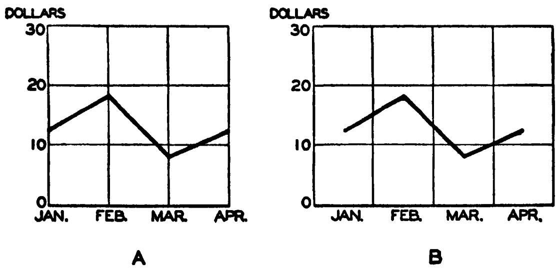

EXHIBIT 78.—Position of points plotted in relation to vertical lines.

use the first month of each quarter (January, April, July, and October), while others prefer to use the last months (March, June, September, and December).[1]

No arrows, dimension lines, or other guides or boxes are necessary to show the relation of months to years. In fact, no lines should be included that are not necessary in reading the chart. In showing data by weeks, the monthly divisions may be confusing if ruled across the chart, but they may be indicated, if

[1] Some statisticians hold that in a chart like Exhibit 76 or 77 the line at the beginning of a space representing a year should carry the figures for December instead of January (as shown). Otherwise it is claimed that the space as a whole does not represent a year. Either plan will serve, however, when it is remembered that in a non-cumulative chart, like Exhibit 76 or 77, the space between lines does not represent a period of time, but the line itself represents a whole month. If the curve is cumulative, then not only the month but the day of the month should be given (see Exhibit 85). Whatever method is used, the chart should be clear to the reader. The practice here should be decided by an international agreement arrived at by the Royal Statistical Society, the American Statistical Association, and other similar organizations. At the present time, however, practice seems to tend toward the establishing of a convention that the line at the beginning of a space representing a year shall carry the figures for January.

desired, by adding short division marks vertically above the zero line (see Exhibit 76).

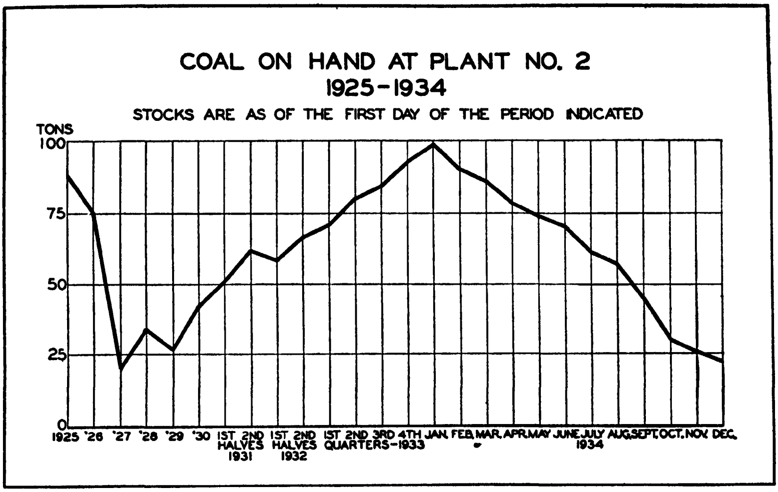

EXHIBIT 79*A*.—Unequal spacing for years. (Incorrect.)

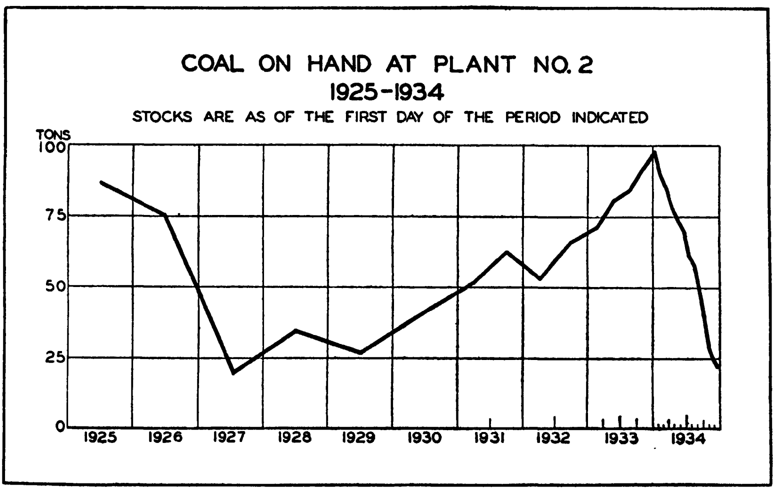

EXHIBIT 79*B*.—Correct method of plotting data of Exhibit 79*A*. (Equal spaces represent equal periods of time.)

Whether points should be plotted on the lines, as in Exhibit 78*A*, or between the lines, as in 78*B*, is a matter to be decided by personal preference. In either case, the designation of the

month (or other period) should be placed exactly below the point plotted in order that there may be no confusion. Thus, in Exhibit 78*A* each name of a month is directly under the vertical *line* on which the point is plotted. In 78*B* each name is directly under the *space* in which the point is plotted.

In the horizontal scale of the usual historical curve, equal spaces should represent equal units of time.[1] In this respect Exhibit 79*A* is wrong and misleading because the spaces on the horizontal scale representing years vary in length. In this chart, the points were plotted annually from 1925 to 1930, inclusive, 1931 and 1932 were plotted semiannually, 1933 was plotted quarterly, and 1934 was plotted monthly. It is not wrong to plot these data in different time units if the years are represented by equal spaces throughout the period, as in Exhibit 79*B*, but it is wrong to give 1934 three times as much space as 1933, six times as much as 1931 and 1932, and twelve times as much as the other years.

If breaks occur in the series being presented, this fact should be indicated graphically. Exhibit 80*A* is incorrect in this respect, as it does not show the distinct break in the series between 1914 and 1919 where 4 years are omitted. If this break is not indicated, the effect is likely to be misleading. The correct method of showing such a break in the horizontal scale is illustrated by Exhibit 80*B*.

Emphasis of Lines.—The zero line should be made heavy in order that it may be sharply distinguished from the other coordinate lines. Usually it is desirable to emphasize the 100 per cent line (Exhibit 76) or any other line that is used as a base for the comparison. In the horizontal scale of a time graph it is usually best not to emphasize the first ordinate since, as a rule, it is no more important than the other ordinates. Sometimes vertical lines representing important time divisions are emphasized for the definite purpose of adding clearness to the graph, but no lines on the graph proper should be emphasized merely for decorative purposes.

The curve on a graph should be heavier than the coordinate ruling, as it should stand out clearly on the background. The curve and the zero line should be prominent, since they define

[1] See previous discussion of the use of the bar chart in presenting irregular or incomplete series (Exhibit 72 on p. 61).

the limits of the sizes represented; that is, the zero line defines the bottoms of the sizes represented and the curve defines the tops. Since our interest is centered chiefly on the curve, it

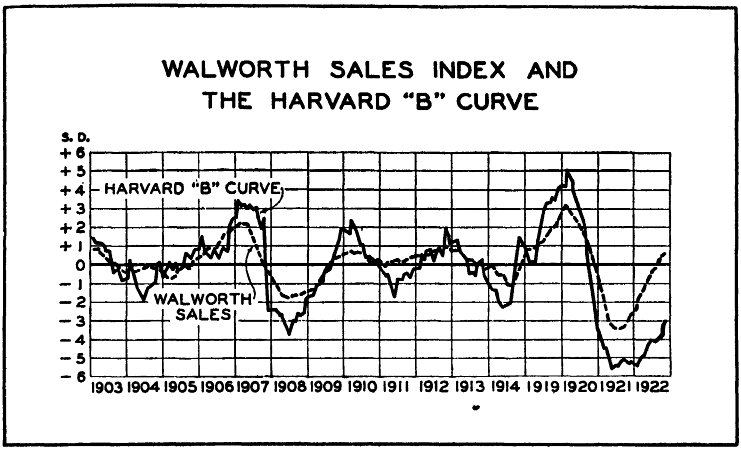

EXHIBIT 80*A*.—Omission in horizontal scale between 1914 and 1919 not indicated graphically. (Incorrect.)

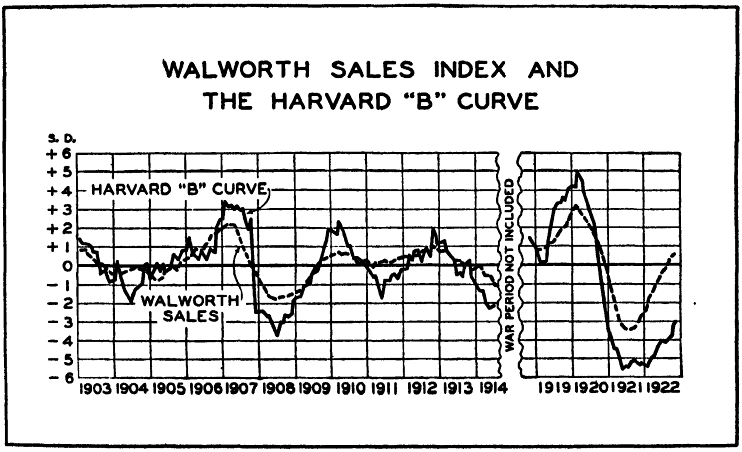

EXHIBIT 80*B*.—Correct method of indicating break in horizontal scale. (Compare with Exhibit 80*A*.)

should be made heavier than the zero line. The background of coordinate lines should be very light as compared with the curves and zero lines (Exhibits 80*B*, 81*B*, and 82).

Scale Units and Number of Coordinate Lines.—The units of the scales should be given in round numbers of intervals con-

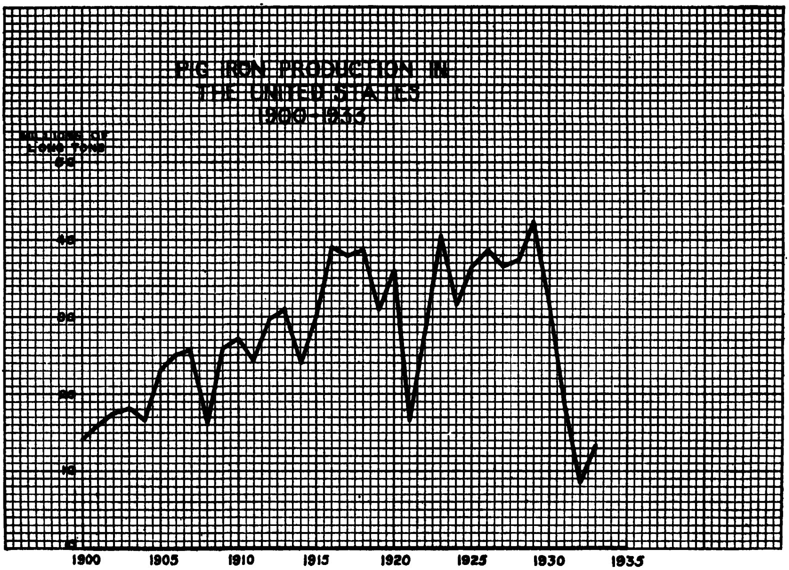

EXHIBIT 81A.—Suitable for working charts, but not suitable for formal presentation.

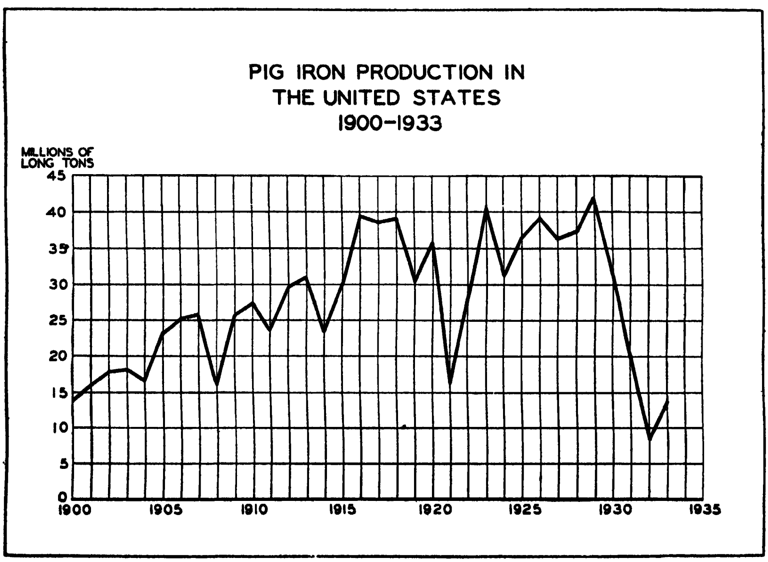

EXHIBIT 81B.—An improved form for showing Exhibit 81A.

venient for reading. No more scale units or coordinate lines should be shown on a graph than are necessary to aid the eye in reading the curve. Just how many to show is a matter for the chart maker's judgment, and will be learned from experience. However, it is easy to see the difference in effect by comparing Exhibits 81*A* and 81*B*. Exhibit 81*B* has all of the lines that are necessary for easy reading, while the large number of lines in 81*A* adds nothing to facilitate making visual comparisons.[1]

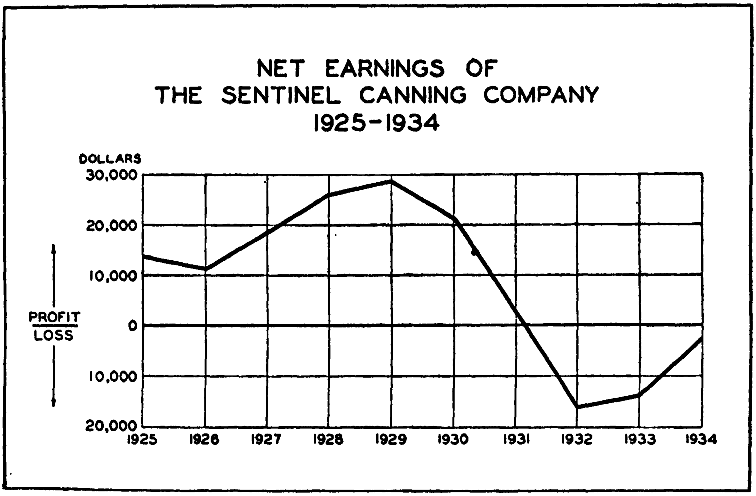

EXHIBIT 82.—Showing positive and negative variations in a single variable.

Showing Plus and Minus Quantities.—Positive numbers are shown above the zero line, and if negative numbers occur in the same series they may be represented along with the positive by plotting below the zero line as shown in Exhibit 82. In the original of this chart (Exhibit 82), profits are in black and losses in red. Blocking in the areas between the curve and the zero base, as in Exhibit 83, makes the chart still more effective for some purposes. While the areas are very irregular (Exhibit 83), the periods of high and low production stand out clearly.

Smoothed and Broken Curves.—In charting non-cumulative data, the plotted points should be connected by straight lines. All that the lines between the points signify is the direction from one point to another.

[1] Exhibit 81*A* was made on coordinate paper, and the closely spaced lines are a convenience in plotting.

For instance, in Exhibit 84 each ordinate represents a month. The first one represents the entire month of January, the second represents February, and so on. There is no time represented between the lines and there is no value on the curve between the plotted points. The point on the February ordinate represents the total for that month, but halfway on the curve between January and February would not represent the sales for the middle of the month. All that the curve does is to point the way from one total to the next. However, considering the period as a

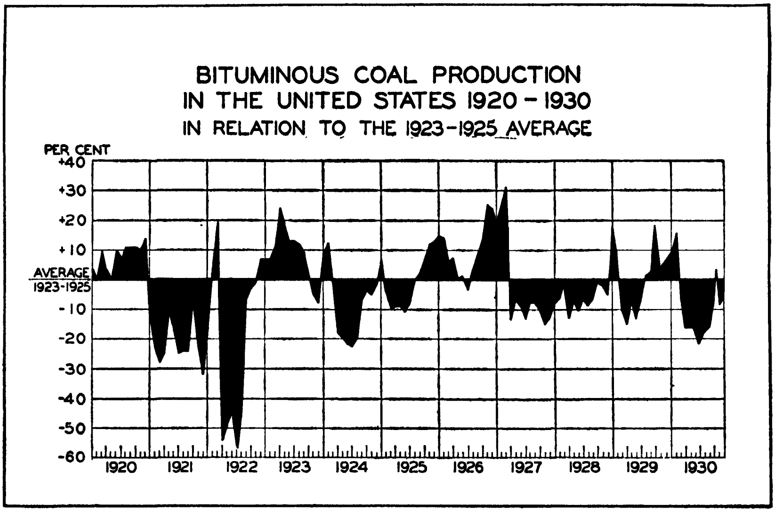

EXHIBIT 83.—Use of shading to emphasize variations. (Compare with Exhibit 82.)

whole, the area under the curve represents the sales over the year.

Exhibit 85 shows the data of Exhibit 84 plotted on a cumulative basis. The space from one month to the next now represents a period of time, and every point on the curve may now be considered as having a value. For instance, the point halfway between January 31 and February 28 represents approximately the sales accumulated from the beginning of the year to the middle of February.[1] In variables such as population series (Exhibit

[1] In laying out the spaces in a scale of months, the differences in their lengths are ordinarily disregarded and the months are equally spaced, as in Exhibit 85.

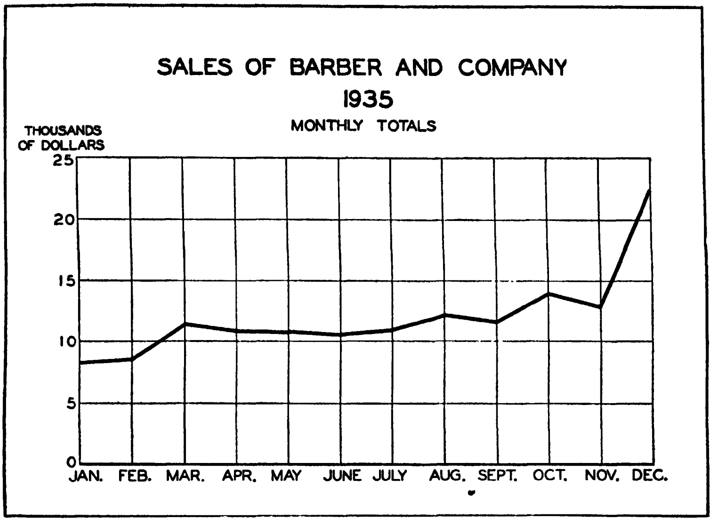

EXHIBIT 84.—Broken curve. (Non-cumulative.)

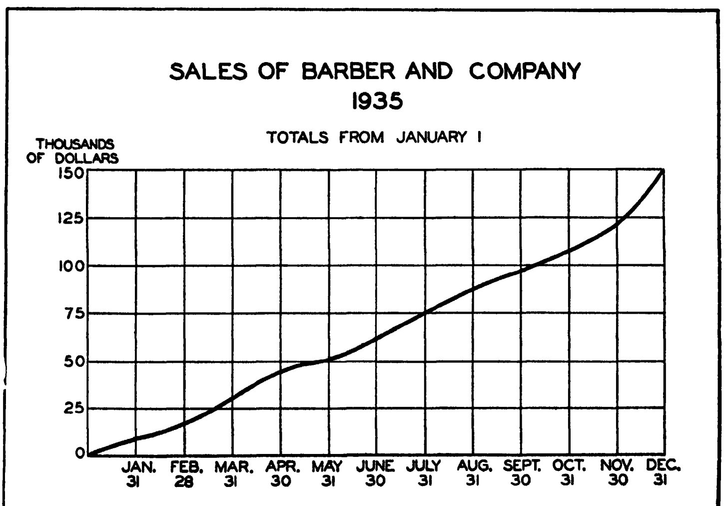

EXHIBIT 85.—Smoothed curve. (Cumulative.)

86), changes rarely occur suddenly, and, therefore, sharp angles are seldom found in curves of such data. Consequently, in plotting cumulative data or a continuous series, the points should be connected by a smoothed curve, as a rule, and not by straight lines. In smoothing a curve, the maximum possible radius of curvature should be maintained constantly, and all sharp breaks should be avoided unless such breaks are actually known to have occurred, as in Exhibit 86 just before the year 1350.

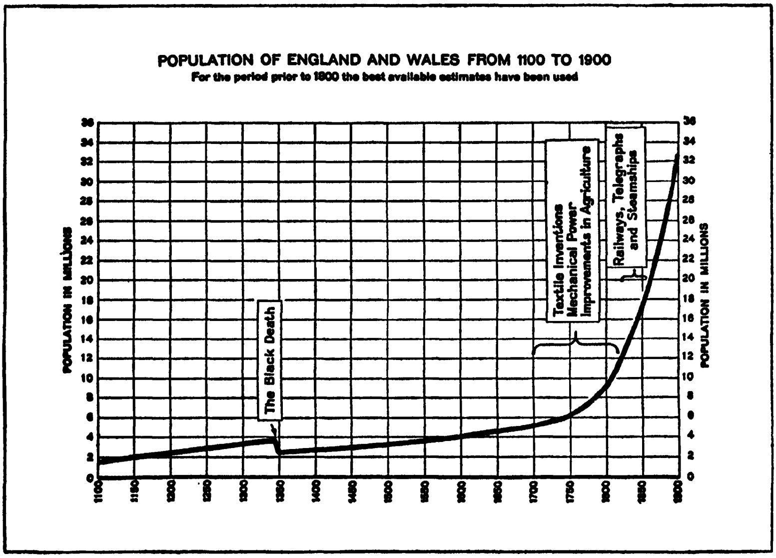

EXHIBIT 86.—Both smoothed and broken curve. (An abrupt break occurs just before 1350. The remainder of the curve is smoothed and represents gradual changes.) (*Courtesy of M. C. Rorty.*)

Throughout this book, curves like those in Exhibit 84 will be referred to as broken curves, and curves like those in Exhibit 85 as smoothed curves. The subject of smoothing curves is discussed in greater detail on pages 146 to 150, inclusive.

The importance of clearly distinguishing between the nature of a cumulative curve and that of a non-cumulative curve cannot be overemphasized. This point will be elaborated upon later in this book in connection with Exhibit 147 on page 133.

Step Curves.—The step curve should be used when changes take place abruptly, as in showing the fluctuations in prices in Exhibit 87 or in number of operators in Exhibit 88. For most

purposes, however, this type of chart is not satisfactory, as it does not show the direction of change over a period so well as

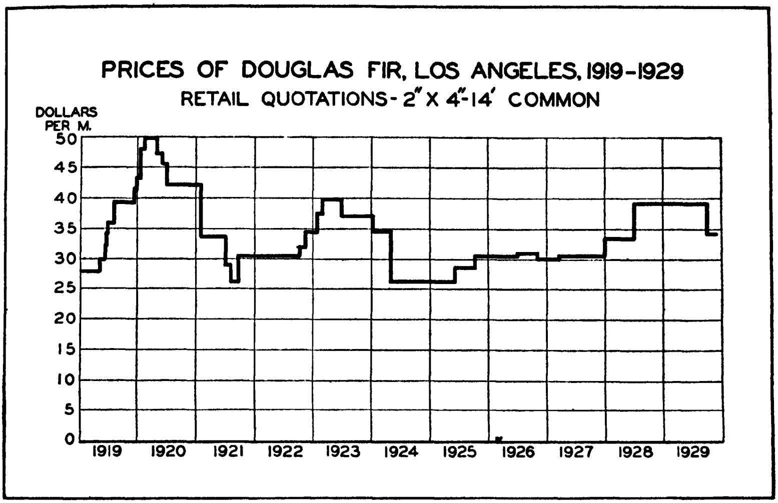

EXHIBIT 87.—A step curve. The step curve should be used when changes occur abruptly but it is ordinarily a poor form for showing the usual series.

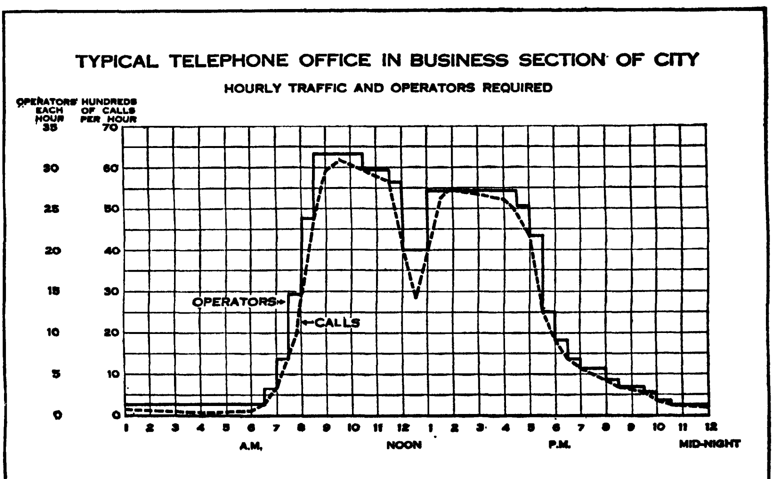

EXHIBIT 88.—The step curve is correct for showing changes which occur abruptly as in the curve of operators in the chart above.

the broken curve, and it does not bring out relations at individual points so well as the bar chart. If two or more step curves

cross one another, they are very confusing, as the vertical parts coincide.

Polar Coordinates.—When the problem is one of presenting data of phenomena which are repeated regularly, as every year or every day, the form shown in Exhibit 89 is sometimes used. While this form may have the advantage of showing a continuous

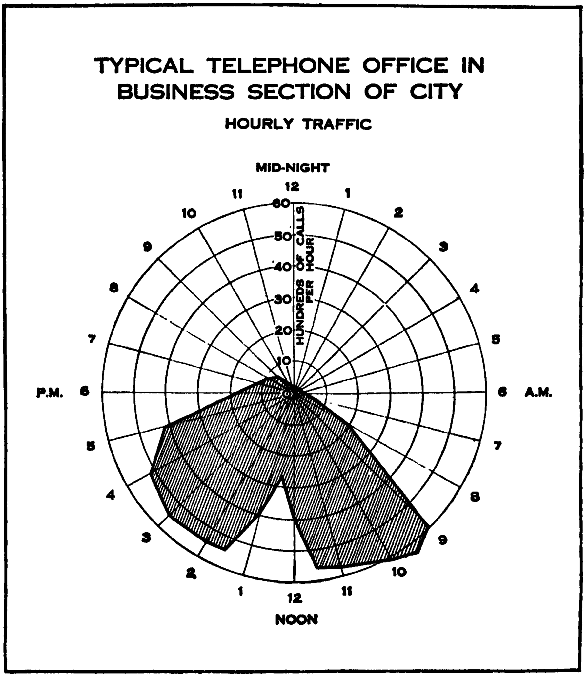

EXHIBIT 89.—For business use the method shown in Exhibit 88 is better than the method shown here. (Compare with the curve of calls in Exhibit 88.)

recurrence better than a curve plotted on rectangular coordinates (Exhibit 88), its use should be discouraged in most instances, as the method used in Exhibit 88 is much easier for the business man to read and understand than the radial plan (polar coordinates) used in Exhibit 89.

An exception to the preceding rule may be made, however, in the case of showing a graph in relation to a clock or other dial instrument. For instance, an effective chart based on the same general plan of Exhibit 89 is shown in Exhibit 90. This method of shading areas, however, exaggerates the values represented.

While occasionally this type of chart (Exhibit 90) may be used effectively, it is not recommended for general use, as the reader is likely to interpret the chart on the basis of area rather than lengths of radii. When the reader is familiar with their uses, polar coordinates are not objectionable as used in connection with various kinds of recording devices, such as voltmeters, ammeters, steam gages, and water pressure gages.[1]

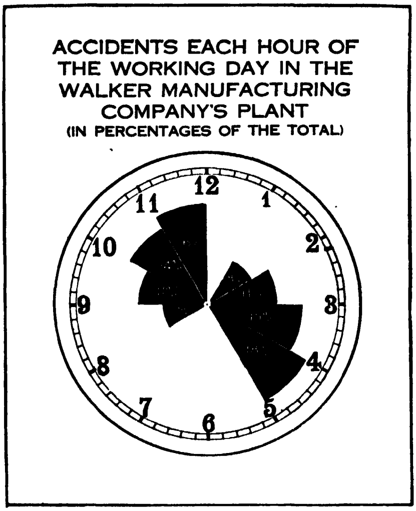

EXHIBIT 90.—The plan of Exhibit 89 applied in popular form to a clock. (Note, however, that the larger values are exaggerated.)

Presenting Tabular Data on Charts.—When it is desirable to present the numerical data along with the graph, this may be done as illustrated in Exhibit 91, in which the data are included at the top of the diagram, or as in Exhibit 92, in which the data are presented as a table. Of course, when the method illustrated in Exhibit 92 is used, the table need not be placed at the end of the graph if it is more convenient to place it elsewhere on the chart. If the table is long or complicated, it is best to present it separately rather than as part of the chart.

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[1] It should be appreciated that, although Exhibits 88, 89, and 90 present time series, these series are not strictly historical. They are presented at this point, however, because of their relation to the historical series under discussion.

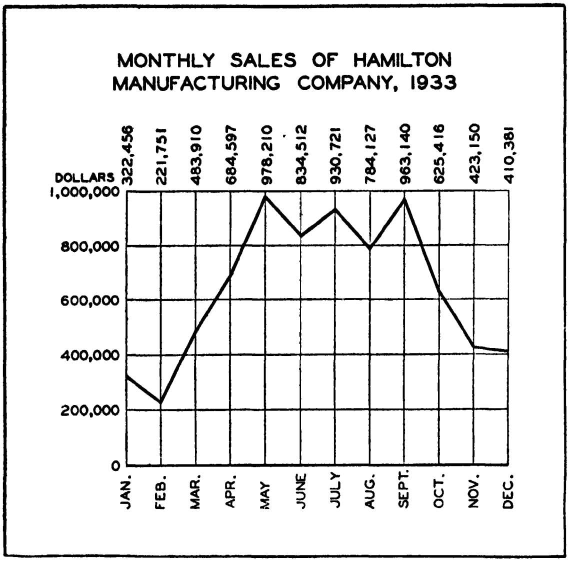


EXHIBIT 91.—Tabulation of data on chart.

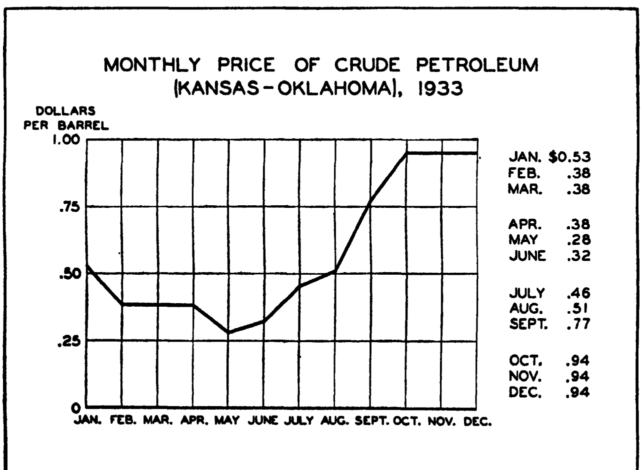


EXHIBIT 92.—Tabulation of data on chart.

COMPARISONS OF SIZE CHANGES IN DIFFERENT TIME VARIABLES

Probably the greatest use of graphic methods in business statistics is, not only in showing how the size of a single variable changes, but in comparing the sizes of two or more variables over the same period of time.

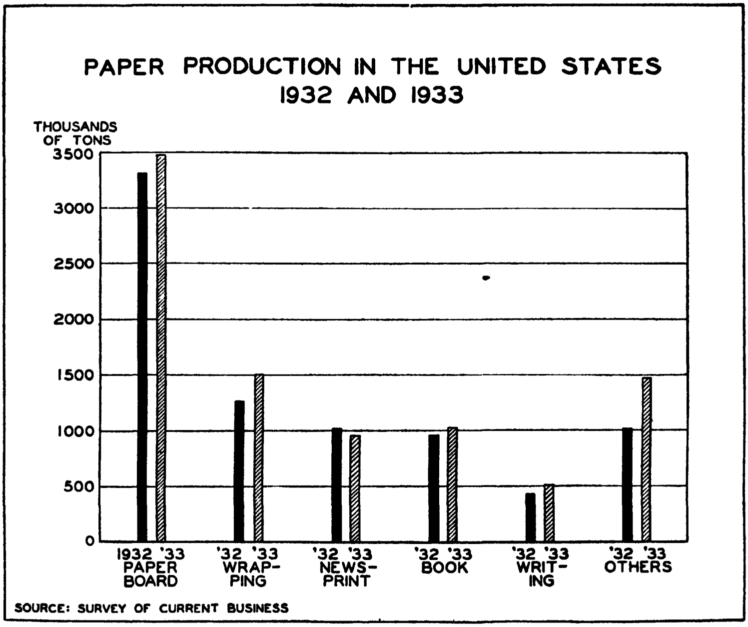


EXHIBIT 93.—Two points of time compared for six pairs of items. (Compare with Exhibit 94.) (*Adapted from Newsprint Service Bureau.*)

Choice of Forms—Bars and Curves.—When, for several items, one point of time is to be compared with only one other point of time, a bar chart like Exhibit 93 is easier to read than a curve chart. In this case, six different items are represented for 1932 and 1933. In order to make the comparison stand out clearly, one kind of shading is used for the 1932 bars and another kind of shading is used for the 1933 bars. Thus, one can easily compare the different items for each year simply by observing the same kind of shading throughout the comparison. When this kind of comparison is made, the bars should be arranged in pairs (or groups) for each item represented, as shown in

Exhibit 93. The time scale for each group should read from left to right as shown. Sometimes the bars of charts like Exhibit 93 are arranged in order of size, but this is not good practice if the normal order from left to right in the time scale is reversed. The correct method is illustrated in Exhibit 93, in that the year 1933 should follow 1932, as shown, and not precede it for the sake of the order of size.

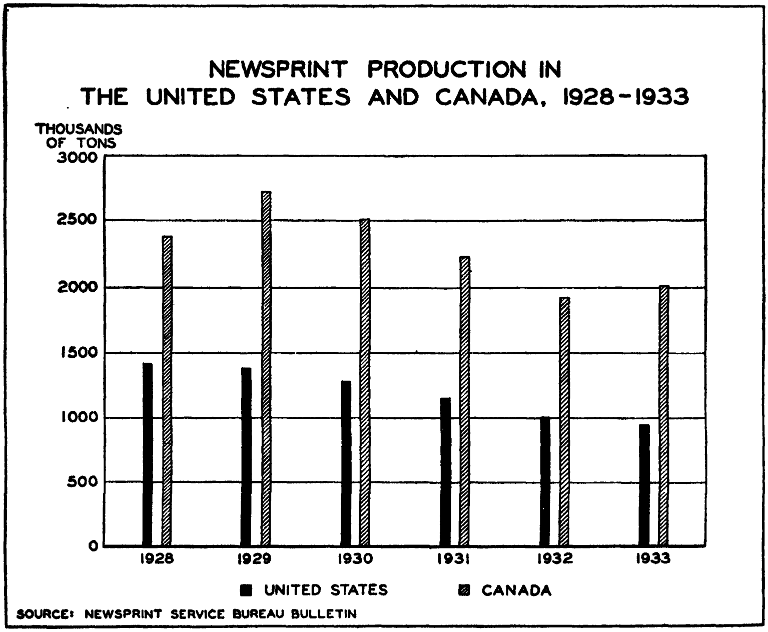


EXHIBIT 94.—Two items compared at six different points of time. Compare with Exhibits 93 and 95.) (*Adapted from Newsprint Service Bureau.*)

In Exhibit 94 a different problem is represented. Exhibit 93 shows 6 items for 2 years, while Exhibit 94 shows 2 items for 6 years. In Exhibit 93 the different shadings represent different *years*. In Exhibit 94 the different shadings represent different *items;* that is, one kind of shading represents United States production over the period, and the other represents Canadian production.

Bar charts like Exhibit 94 may not be easy to read if more than two series are represented. Three, four, or more different kinds of shaded bars for each year are difficult to follow over a

period, although there are instances when the comparisons within each group are so important that the use of several bars for each period is justified (see later discussion of Exhibit 96). While a chart like Exhibit 94 presents the comparison quite effectively, especially when only two series are to be shown, the curve method is most commonly used in such presentations. Exhibit 95 shows the data of the bar chart in Exhibit 94 plotted

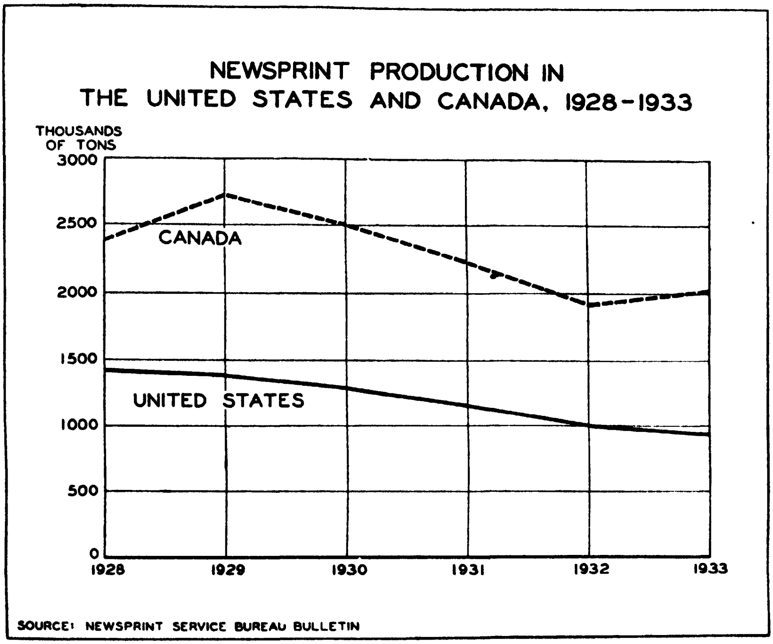


EXHIBIT 95.—Data of Exhibit 94 presented as curves. (The most common method of presenting this kind of comparison.) (*Adapted from Newsprint Service Bureau.*)

as arithmetic curves. Often the bar-chart method of Exhibit 94 is better than the curve method of Exhibit 95 when data are presented to those inexperienced in the use of charts, but otherwise both charts are read in the same way and give the same impression.

Exhibit 96 presents an example of a chart in which groups of four bars are shown for each year (upper graph) and for each month (lower graph). While it is more difficult to compare the series over the entire periods shown than it would be if curves were used, the relations within each group are sufficiently

important to justify the bar method. The bars in each group show the relations between production, shipments, stocks, and

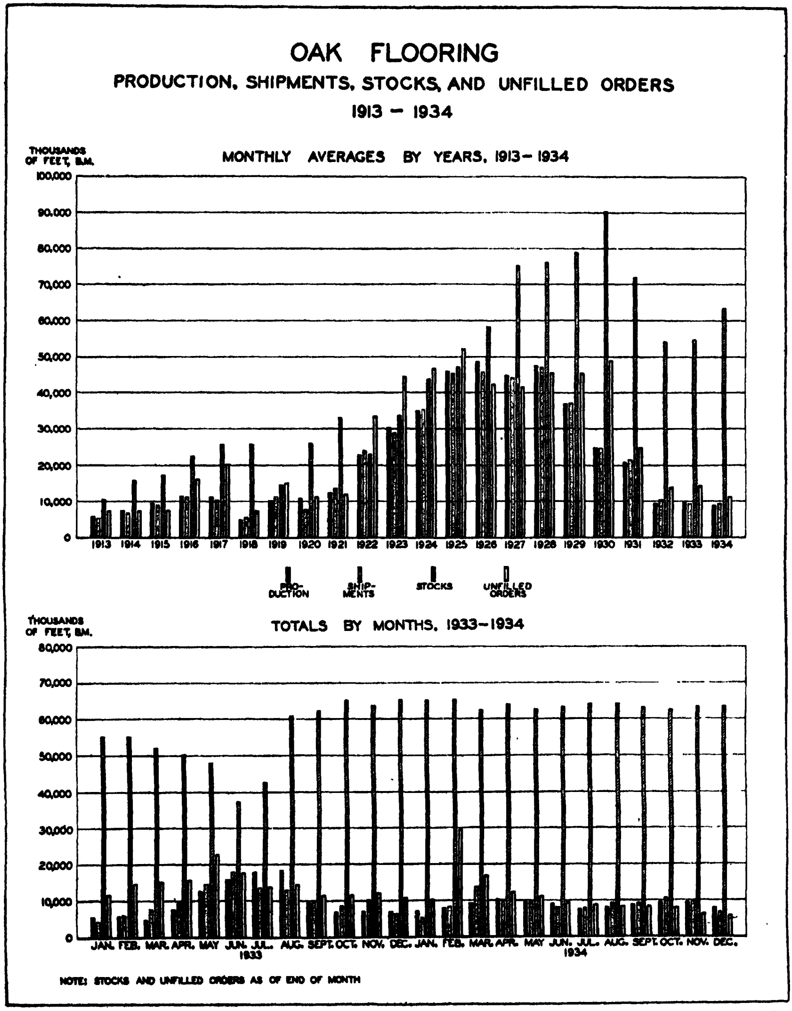


EXHIBIT 96.—Bar chart comparing four items for each unit of time in the periods represented. *(Data from Survey of Current Business.)*

unfilled orders in such a way that the business man concerned can plan his own operations with due regard for the conditions shown.

Scales for Size Comparisons.—In Exhibit 94 we compare the lengths of bars representing the United States with the lengths of those representing Canada. In Exhibit 95 we compare the space between zero and the curve for the United States with the space between zero and the curve for Canada. Such comparisons indicate how important it is to begin with zero and to show the whole vertical scale. The importance of this principle is indicated further by Exhibit 97, which (without the dotted addition) is likely to leave the visual impression that exports in 1924, for instance, were about twice as great as imports, while, as a

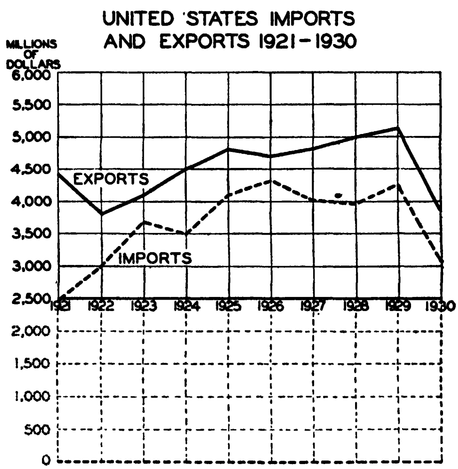


EXHIBIT 97.—In comparing sizes it is important to include the whole scale.

matter of fact, by studying the scale one can see that they were greater by less than one-third. This true relation would be shown clearly if the chart were drawn to include the entire scale as indicated by the dotted lines. If for some reason the entire scale is not used, the reader should be warned as explained in connection with Exhibit 75 on page 64.

Variables of positive and negative values may be compared by plotting above and below the zero line as shown in Exhibit 98. Note that the shading of the resultant tonnage is especially effective.

When comparing sizes, it is necessary to have a scale that is common to all of the series considered. That is, we cannot compare cotton in bales with wool in pounds and get a true impression of the relation between the amounts of the two textiles.

This can be done only by using a common unit, say, pounds. We cannot directly compare quantities of oil in barrels with tons of coal, but, by reducing them to pounds, monetary values, heat units, or other units common to both, we can plot them to the same scale and make direct comparisons.

A double-scale arithmetic chart, like that in Exhibit 99, is not recommended for general use. It is likely to mislead the reader, who might assume that the curves may be compared directly,

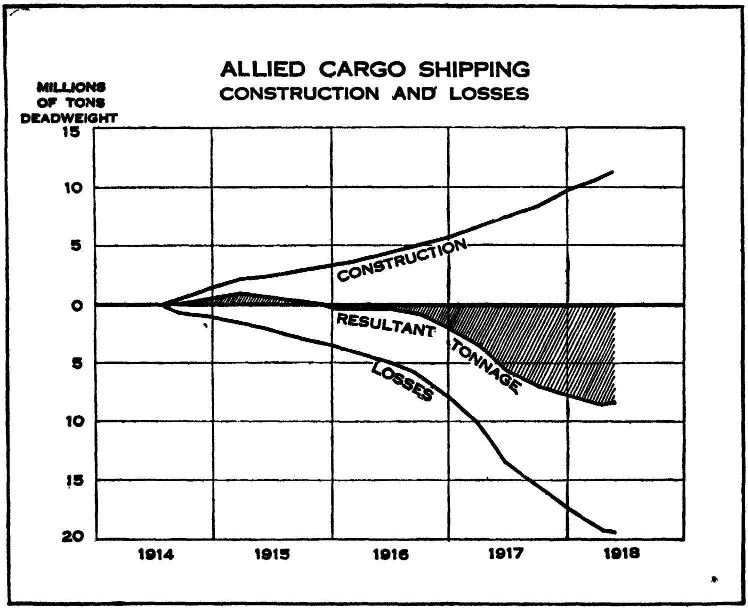


EXHIBIT 98.—Comparison of positive and negative values. (*Adapted from General Electric Co.*)

since they are placed on the same graph. Such curves, as a rule, should be plotted to a common scale or on separate graphs. If the *sizes* of the items are to be compared over the time period, it is imperative to have a common arithmetic scale. If *rates of change* are to be shown, the curve should be plotted as described in the following chapter.

Although it is not good practice, as a rule, to use more than one arithmetic scale on a graph, situations occur occasionally which make this procedure desirable. Exhibit 88 on page 75 is an example of the use of two different arithmetic scales and, since there is a definite relationship between the two, a good

comparison can be made. When two scales are used, both should be placed on the same side of the graph (as in Exhibits 88 and 99) rather than one on each side. If a different scale is placed on each side, the left-hand ends of the curves are liable to be read from the left-hand scale, and the right-hand ends from the right-hand scale.

Differentiating Curves.—When two or more variables are represented on a graph, it is necessary so to differentiate the

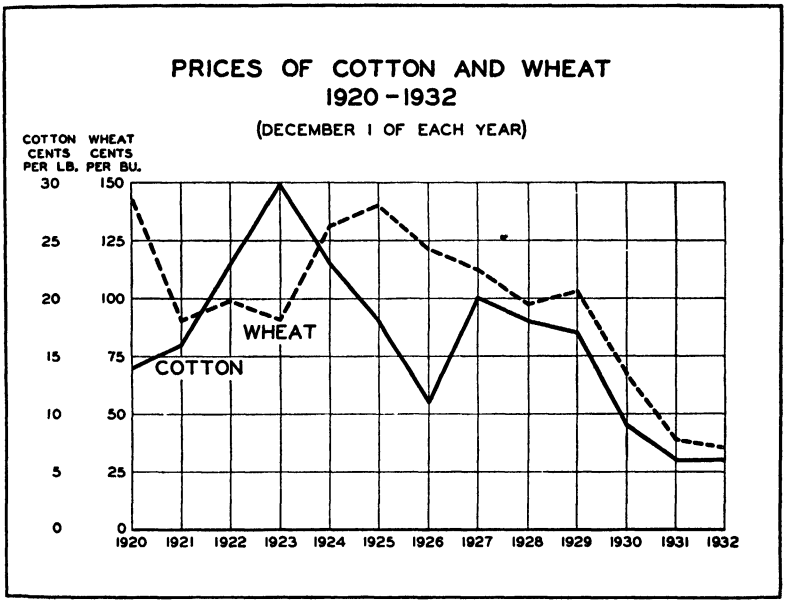
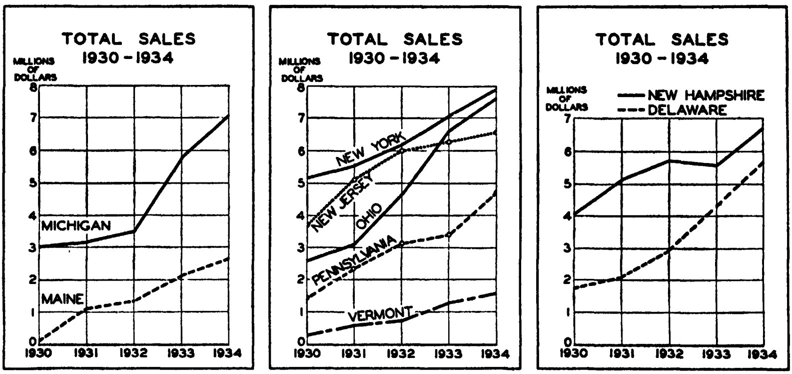


EXHIBIT 99.—Such a use of more than one scale ordinarily is bad practice. The curves cannot be compared as to sizes represented.

curves that there will be no difficulty in distinguishing them. This practice is desirable even if the curves are in different parts of the field and do not cross, especially if there is any likelihood that later periods or other series may be added. If the curves are close together and cross each other, it is particularly necessary that they be made in such a way that each curve can be followed across the graph without confusion. There are two common methods of differentiating curves. One is by using different colors, and the other is by using different kinds of dot and dash combinations in black such as those illustrated in Exhibit 100. The latter method is the one commonly used when

the charts are to be reproduced by any kind of photographic process.

Designating Curves.—When two or more different curves are plotted on a graph, it is necessary to designate them with the names of what they represent. This is done in two different ways. One is to place the names along the curves near the left-hand ends or beginnings of the curves (Exhibits 100*A* and *B*). This method is most convenient from the point of view of the reader, since the designation is directly connected with the curve. The designations are lettered horizontally if there is room



A *B* *C*

EXHIBIT 100.—Differentiating and designating curves.

(Exhibit 100*A*); but if the curves are crowded, the designations may be lettered along the curves as in Exhibit 100*B*. The other method is to designate the meanings of the different kinds of lines by a separate key or legend elsewhere on the chart as in Exhibit 100*C*. Of these two methods, the one illustrated in Parts *A* and *B* is recommended as best for general use.

Number of Curves.—The number of curves that can be placed successfully upon a single graph depends chiefly upon the way in which they are distributed over the field, but in no case should as many curves be plotted on one graph as are shown in Exhibit 101. In such a chart it is almost impossible to follow a curve across the graph. In Exhibit 102 seven curves are too many, as they run together so badly that the individual curves can be distinguished only with difficulty. An exception may be made to this rule, however, when curves are plotted together to show correlation, which is the purpose of Exhibit 102.

PRODUCTION OF BITUMINOUS COAL IN MICHIGAN AND STATES WEST OF MISSISSIPPI RIVER
1917 - 1918

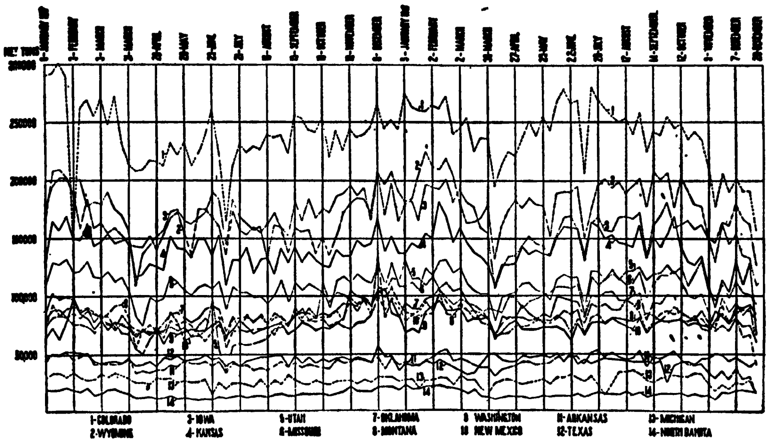


EXHIBIT 101.—Hopeless confusion. Too many curves. (Other errors are light zero line and wrong position of time scale.)

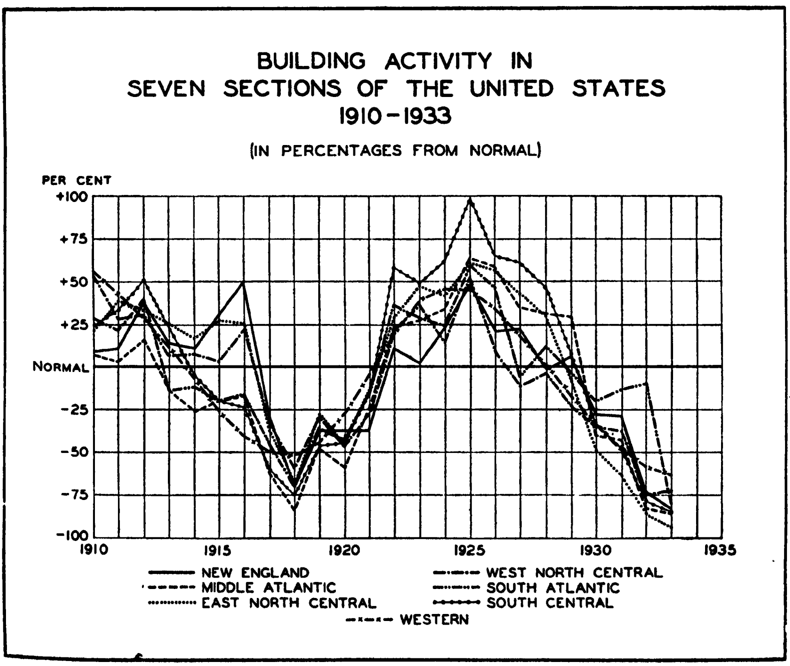


EXHIBIT 102.—Confusion of individual curves, but good practical method of showing correlation.

If a large number of curves must be shown on a single chart, the method illustrated in Exhibit 103 may be used which shows twenty curves successfully. A large number of curves may be shown clearly and compared easily by using this method of

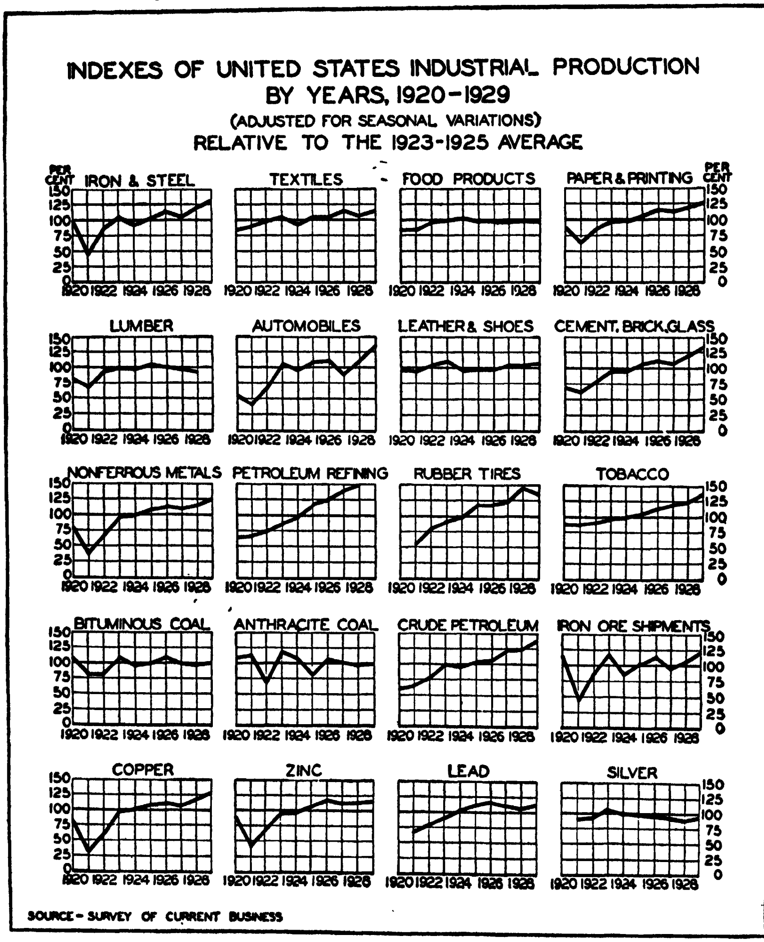


EXHIBIT 103.—A good method of showing a large number of curves.

dividing a chart into a number of small graphs which, if plotted as a single graph, would result in hopeless confusion. When charted as in Exhibit 103, all of the small graphs should be drawn to exactly the same scales. Otherwise, they cannot be compared easily and they might be compared inaccurately.

Standardization.—Annual figures, or data for other definite periods, are often placed on different charts for convenience, and, when it is desired to make direct comparisons of these different charts, they should be drawn to the same scales so far as possible. That is, the units of the vertical scales should be the same on all the charts, and the units of the horizontal scales should be the same on all the charts. For instance, if 1 inch equals $100 in the vertical scale of one chart, it should equal the same amount on each of the other charts, and if 1 inch equals 1 year in the horizontal scale of one chart, it should equal the same period on each of the others. Without this standardization, the curves may be distorted, as illustrated in Exhibit 104, which shows how

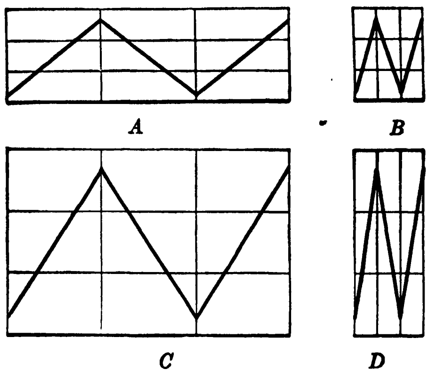


EXHIBIT 104.—Distortion caused by different scales.

exactly the same data appear when plotted to different kinds of coordinate spaces. Comparison of sizes over a period of time, therefore, is greatly facilitated by the use of common scales, whether the comparisons are made on the same or on separate charts.

In Exhibit 105 arithmetic curves have been laid out proportionally in an attempt to show relative change. That is, the ranges of the variations in the two series are given about the same space in the vertical scales of both graphs, and the units are designated accordingly, which is in a ratio of one to two. Thus, if the "New York" curve drops from 1,000 to 500, it is a decrease of 50 per cent. If the "All Others" curve drops from 2,000 to 1,000, it also decreases 50 per cent. These changes would be represented by the same distances on the scale when

they are laid out as shown in this example. This practice, however, is not advised for general use, as it is best to use the logarithmic chart when showing rates of change.[1]

Range of Variations.—In Exhibit 106 curves are plotted for both high and low prices for each series and the areas between are shaded. Thus, the range of stock prices over the period is clearly shown. Another good method of representing range of prices is shown by Exhibit 107. Here, the top of the bar repre-

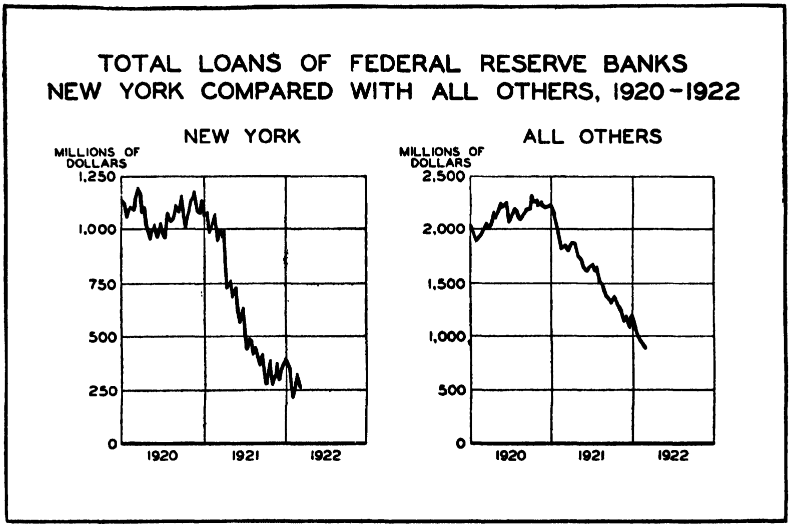


EXHIBIT 105.—Scales laid out in proportion to range. (Not advised for general use—see Exhibit 145 on page 130.)

sents high and the bottom represents low, while the length shows the range. This method enables the ranges for different dates to be compared readily.

Double Time Scales.—Exhibit 108 illustrates the use of a double time scale in the comparison of two separate periods. Here two different sections of time are compared, but the units are not different, as yearly intervals are used for both curves. In this way, one year (plotted in months or weeks) is often compared with another in studying seasonal variations (see

[1] For further explanation of the dangers involved in this practice, see the discussion of Exhibit 145 on pp. 129–131.

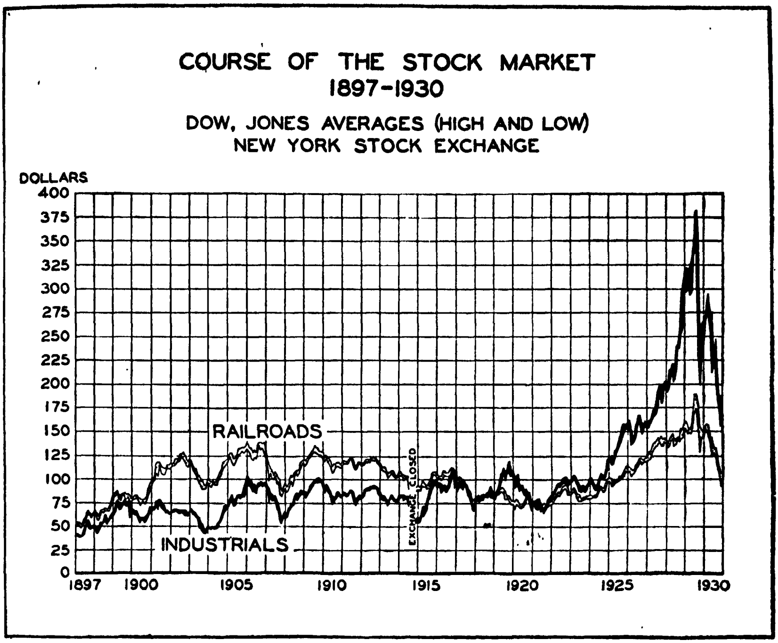


Exhibit 106.—Range of variation. (Shading between curves.)

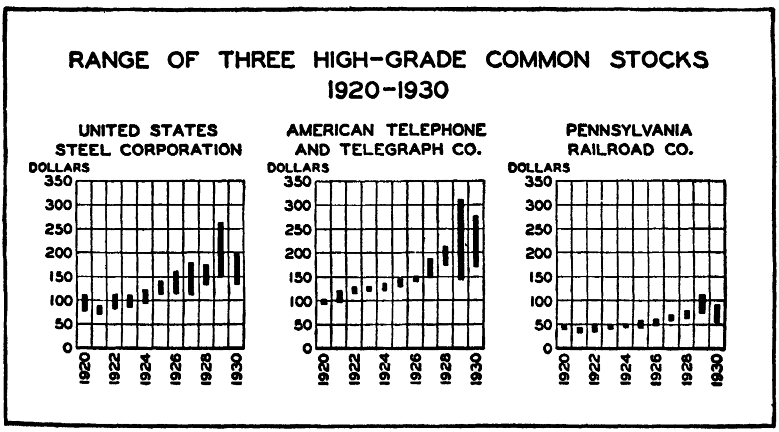


Exhibit 107.—Range of variation. (Bars.)

Exhibit 139 on page 123). The curves in Exhibit 108 are freely smoothed from the points plotted on the vertical lines.

Different Time Scales.—Often it is desirable to show the more current data in greater detail than the more remote. Thus, in Exhibit 109 the current year is shown on a monthly basis, while the preceding years are shown on an annual basis. Two different time scales are used. When data are shown in this way, it is important for the sake of clearness and accuracy to leave a substantial space between the two graphs, as shown in Exhibit 109.

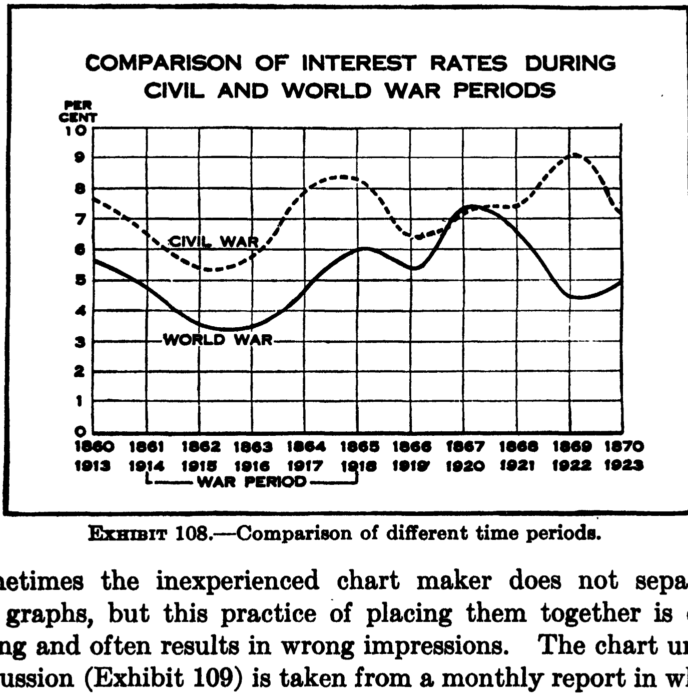


EXHIBIT 108.—Comparison of different time periods.

Sometimes the inexperienced chart maker does not separate the graphs, but this practice of placing them together is confusing and often results in wrong impressions. The chart under discussion (Exhibit 109) is taken from a monthly report in which the same form is used each month. The original form is laid out at the beginning of the year and then, as the data for each succeeding month become available, the chart is brought up to date and a photostat copy is made of it.

Smoothed and Broken Curves in Comparisons.—The rule for choosing between smoothed and broken curves in comparisons is, of course, the same as previously discussed for the single curve. If the value represented for each period does not include any of the value for the previous period, all that the curve shows

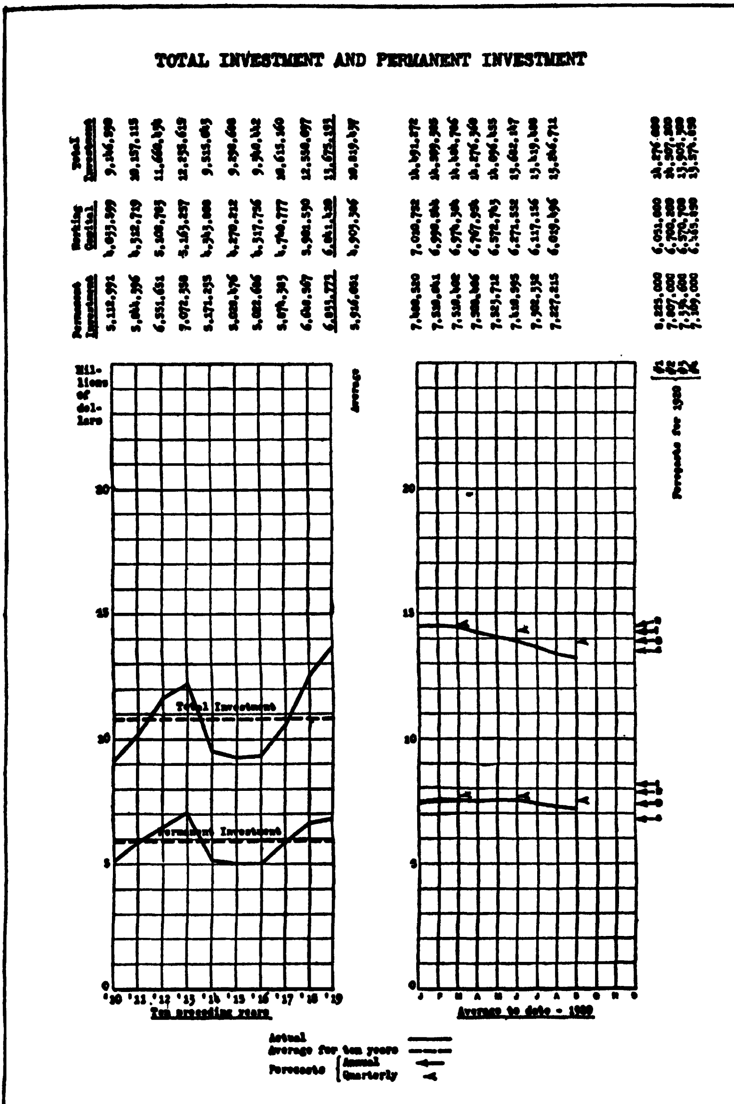


EXHIBIT 109.—Different time scales. Separate graphs for showing remote and current data together on one chart. (*Courtesy of E. I. du Pont de Nemours & Co. Data hypothetical.*)

is the direction of the change in the new figure, that is, whether or not it is greater or less or the same. The values are represented by the plotted points, and there is no value on the curve between points. Thus, in the case of the monthly sales curve in Exhibit 110 (solid line) the point plotted on the January ordinate represents the sales for January, the point plotted on the February ordinate represents the sales for February, and so on. The ordinate represents the month, and the space between the lines does not represent any more than does the space between bars when comparing, say, populations of New York and Chicago.

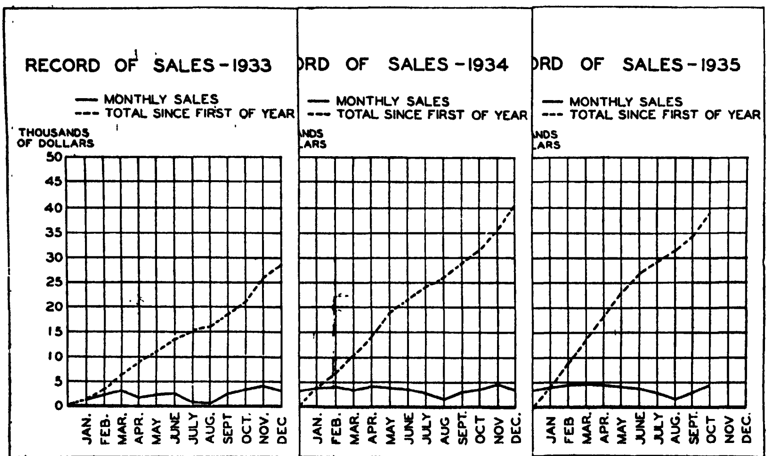


EXHIBIT 110.—Use of separate cards for showing continued series.

The only purpose of the curve in showing non-cumulative data over a period of time is to indicate the direction from one point to the next, and to define the area between the points and the zero line; this area is used as a basis in comparing the different series.

If, however, the value at any point includes the value represented at the preceding point and is merely a change in the total, or if the new value is simply a variation in the old value, there seldom will be abrupt breaks, and a smoothed curve is ordinarily more representative than a broken curve. Thus, the smoothed curve was used in Exhibit 108 because the old rate changes—there is not an entirely new rate established each year. In Exhibit 110 the curve for "Total since first of year" (dotted line)

is cumulative and is gradually changing throughout the space between the plotted points. It follows, therefore, that the point on the dotted line midway between the lines representing June 30 and July 31 would give, very closely, the total accumulated from the first of the year to July 15.

When curves are smoothed as freely as those shown in Exhibit 108, they are sometimes misleading to those who do not understand them. In common practice, such curves are usually plotted as broken curves.

Since, in charting cumulative figures, a period of time is represented by the space between the designated dates, it is necessary to have as many *spaces* as there are plotted points in such charts as Exhibit 111. In this chart, the figures are plotted on the last of each month. In order to have the chart complete, a space must be allowed at the beginning of the graph to show the accumulation during the month of January. In annual graphs of non-cumulative figures, twelve lines (eleven spaces) are all that are needed if each line represents a month; but when such charts are drawn to be compared with other charts, as in Exhibit 110, it is well to have an extra space at the beginning or the end of non-cumulative graphs in order that the space between January and December may be the same as between the other months when the graphs are placed end to end (see also Exhibit 5 on page 11).

Shading to Show Differences.—The use of shaded areas, as in Exhibit 111, often brings out certain facts more vividly than the use of curves alone. In Exhibit 111 the dashed line represents the materials consumed in manufacture. The dotted line represents the figures at which the raw material inventory must stand if there is always to be a 4 months' supply on hand. It will be noted that a certain point on the dotted line is reached by the dashed line just 4 months later. The solid line represents the actual inventory on hand. In order to emphasize the difference between the actual and the ideal supplies, the area between the two lines was shaded, and this brings out very clearly the excess of unnecessary raw materials.

A method similar to the foregoing is used in Exhibit 112, in which the solid line represents an ideal expense budget of $100 per month. The dashed line shows the actual expenditure to date and how much it is above or below the ideal. In the original

of this chart the areas above the solid line were shaded in red and those below in black. This method of shading is very

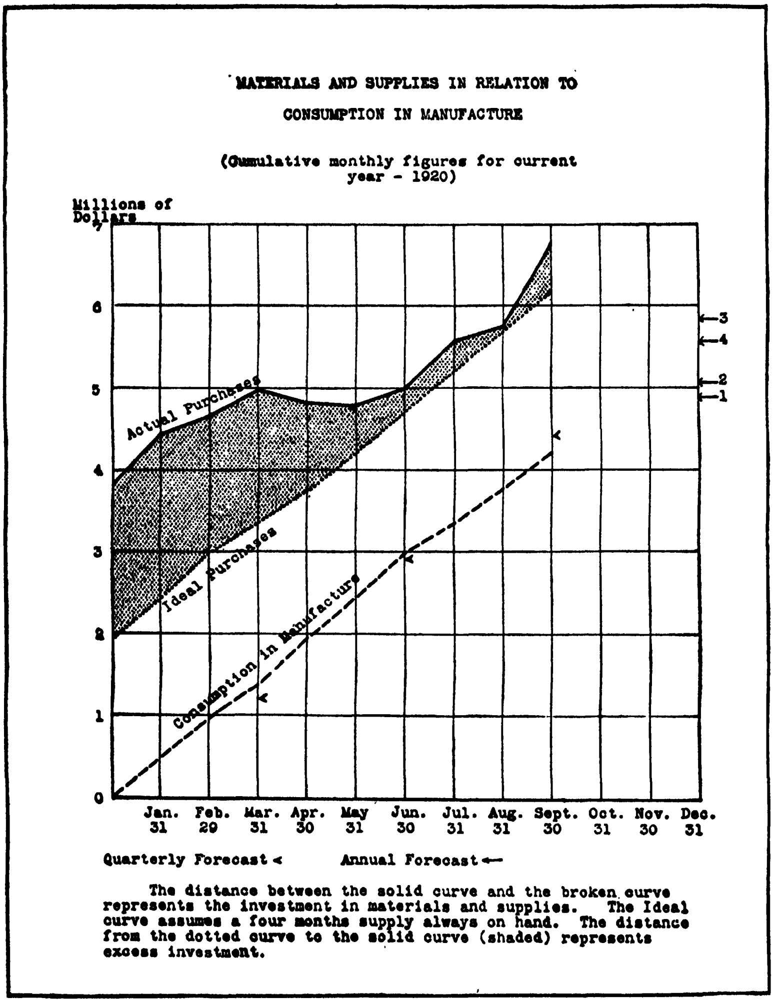


Exhibit 111.—Shading to emphasize differences. (*Adapted from E. I. du Pont de Nemours & Co. Data hypothetical.*)

effective in calling attention to whether the expenditures are too large or too small.

Indicating Forecasts.—The arrows on the chart in Exhibit 111 represent forecasts. At the beginning of the year a forecast is

made for the manufactures curve for 3 months, and this point is indicated by the arrowhead at March 31. At the same time the point which the curve will reach at the end of the year is estimated, and indicated by arrow 1 at the end of the graph. The quarterly forecasts are made at the end of each quarter, and at the same time the forecast for the end of the year is revised, and indicated as shown. Other forecasts, such as a moving

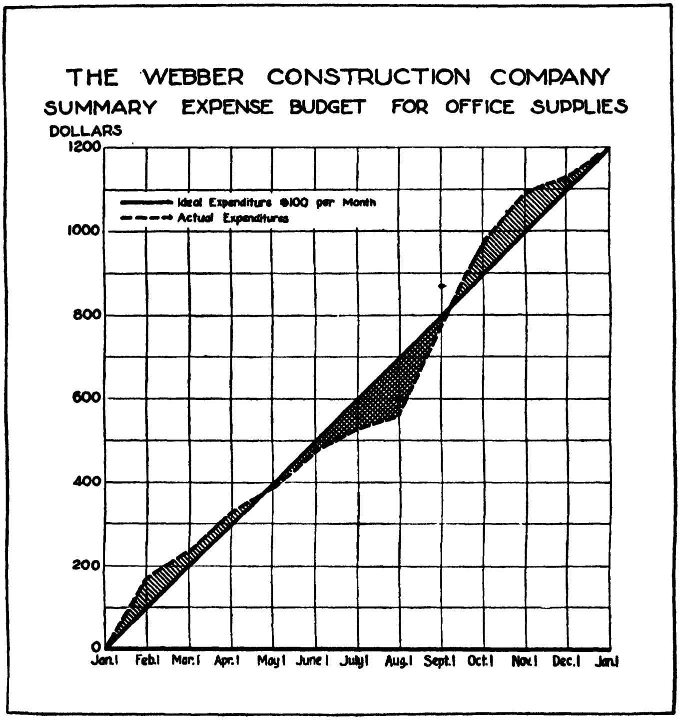


EXHIBIT 112.—Shading to emphasize differences between ideal and actual expense.

annual forecast, may be indicated in the same way. Another method of indicating forecasts is to use separate curves.

COMPONENTS OF A TIME VARIABLE

Choice of Forms—Bars and Curves.—Often it is desirable to show the component parts of a time variable and this may be done in various ways. Exhibit 113 illustrates a simple method of showing how new passenger-car registrations in the United States

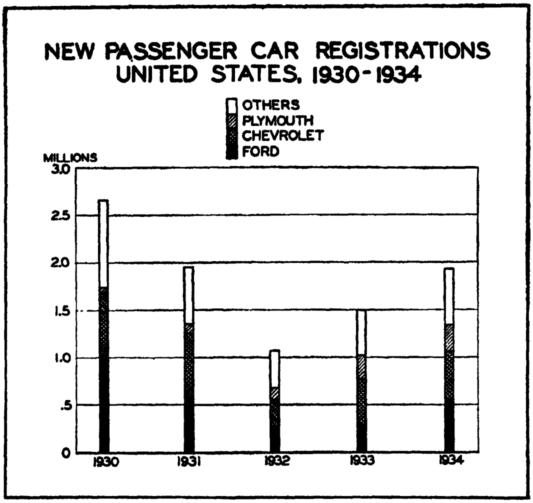


EXHIBIT 113.—Component parts of a time variable shown by separate component bars. (*Adapted from Standard Statistics Service.*)

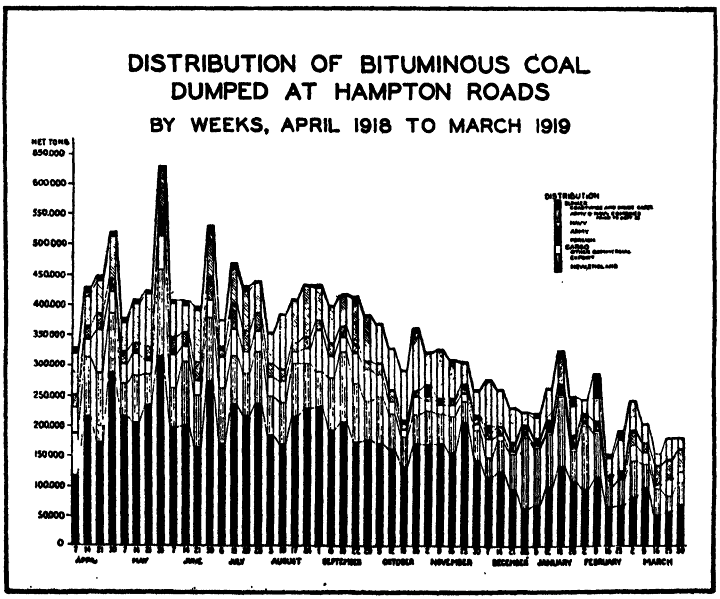


EXHIBIT 114.—Component bars connected by lines.

were distributed by makes over the period from 1930 to 1934. In Exhibit 114 the lines connecting the components of the bars are intended to aid in following a certain component over the period. However, this chart is somewhat difficult to read, as the

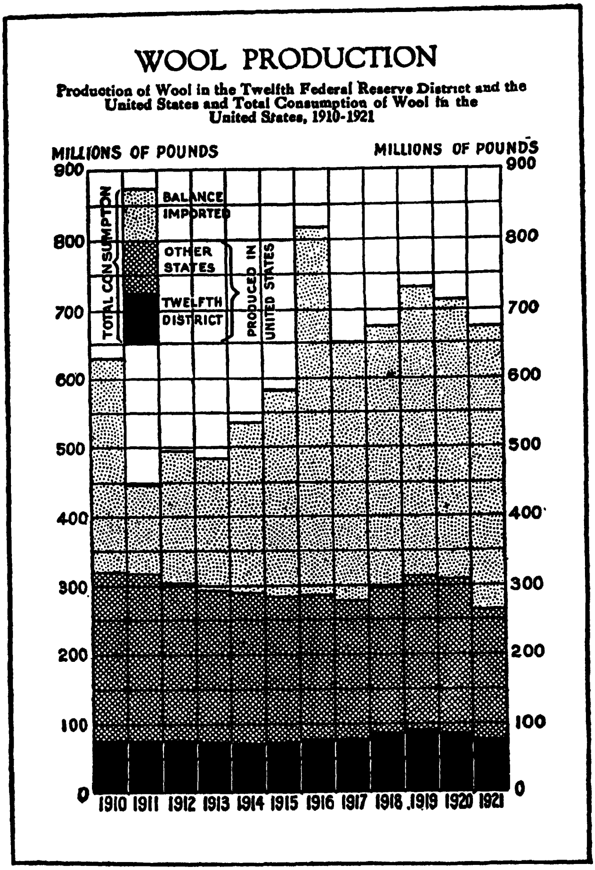


Exhibit 115.—Component bars with no intervening spaces. (*Courtesy of Federal Reserve Bank of San Francisco.*)

cross-hatching is not distinct and there are too many small components to allow clean-cut comparisons. In Exhibit 115 the chart is easy to read, as the components are continuous over the period and can be followed directly across the chart. This condition is possible, however, since the data do not change much, and the same kinds of components are always opposite

each other. That is, for instance (in Exhibit 115), an "Other States" component is never opposite a "Balance Imported" component. This difficulty is often met in Exhibit 114, which would be easier to follow over the period if it were plotted as curves and the areas between the curves were shaded as in Exhibit 116. Exhibit 115 should be considered as bars placed together—not as step curves with the areas filled in between

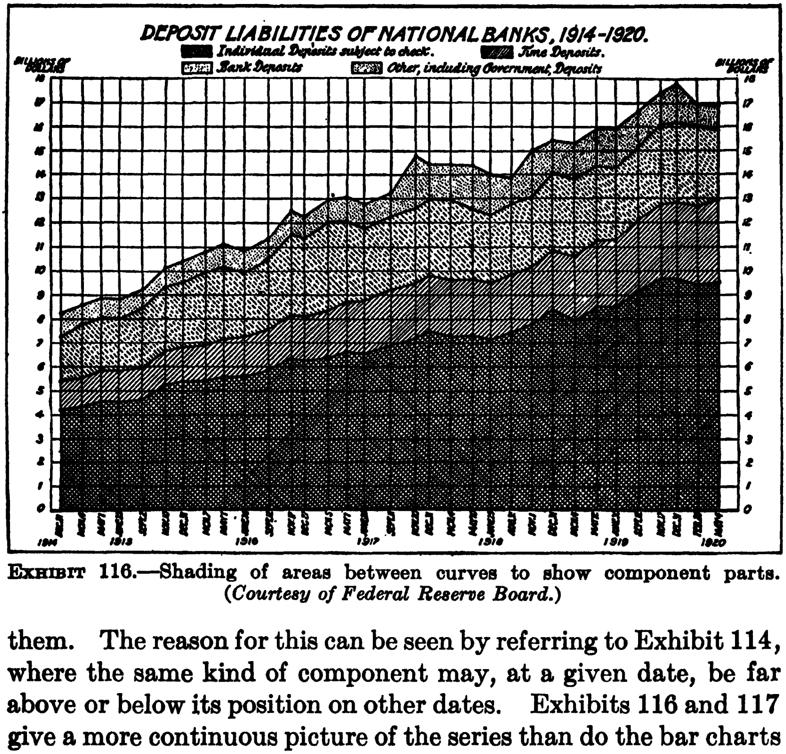


EXHIBIT 116.—Shading of areas between curves to show component parts. (*Courtesy of Federal Reserve Board.*)

them. The reason for this can be seen by referring to Exhibit 114, where the same kind of component may, at a given date, be far above or below its position on other dates. Exhibits 116 and 117 give a more continuous picture of the series than do the bar charts in Exhibits 113, 114, and 115.

Designating Shaded Areas.—Two methods of designating the "belts," as the areas between the curves are called, are illustrated by Exhibits 116 and 117. If there is room for the designations, the method of placing them on the belts, rather than in an outside legend or key, makes them easier to read. Large white spaces should not be blocked out in the belts for the designations, as this interferes with the conception of the sizes of the areas.

Actual Sizes and Percentages.—So far, this description of component parts of time variables has included only actual sizes of the values represented. Frequently it is desired to know what per-

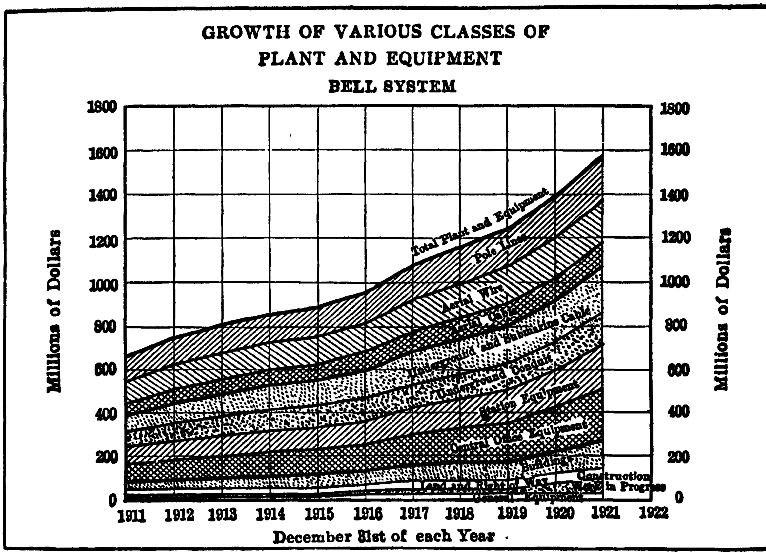


EXHIBIT 117.—Designation of belts on a component time chart. Compare with legend on chart in Exhibit 116. (*Courtesy of American Telephone and Telegraph Company.*)

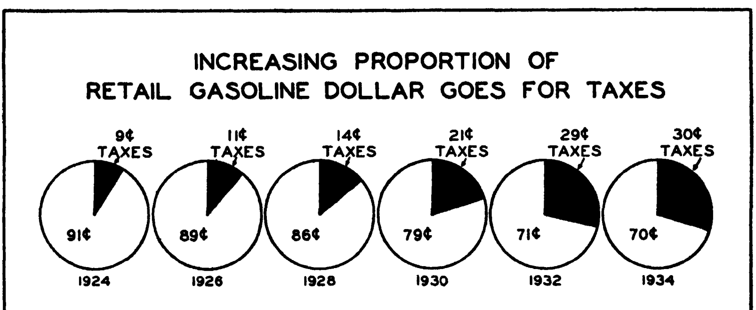


EXHIBIT 118.—A simple method of showing changes in component parts over a period of time. (*Adapted from Automobile Manufacturers Association.*)

centage changes have taken place without regard for the actual sizes of the changes. In instances where the total does not change in size over a period, the actual size distribution and the percentage distribution of the components would appear the same when

shown graphically. When the totals vary in size over the period, they may be represented equally as 100 per cent, and the components plotted proportionally.

A component time chart based on a series of "pie" diagrams is presented in Exhibit 118. In this illustration each circle represents 100 per cent and the sectors represent the components.

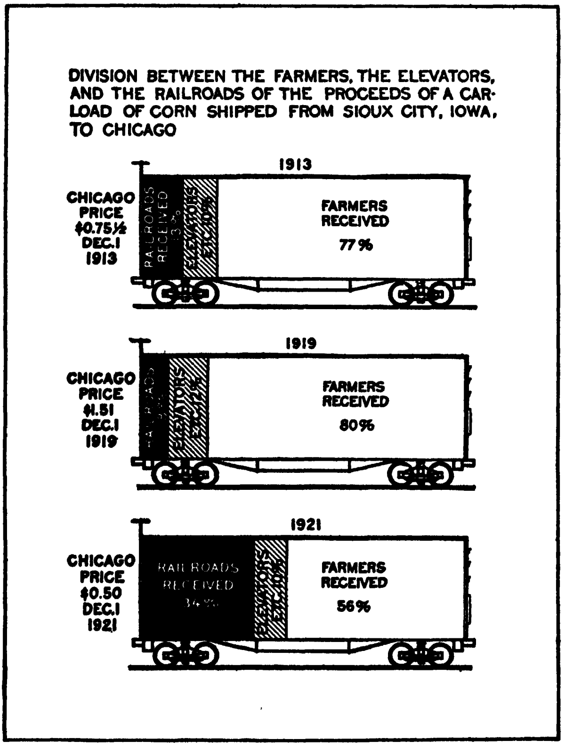


Exhibit 119.—A pictorial component time chart. (*Courtesy of the United States Department of Agriculture.*)

The circles are equally spaced and the spaces represent equal intervals of time. The chart is clear and simple and would be understood readily by readers of popular publications.

Another simple chart that is understood easily is shown in Exhibit 119. Pictorial methods of this type are valuable in making popular presentations and offer an almost unlimited field of possibilities.

It is easier for the average reader to grasp the relative values of Exhibit 121 than of Exhibit 120. A conception of the relations

in Exhibit 120 can be gained with less effort if but one curve is used and the areas are shaded as in Exhibit 121. The shading

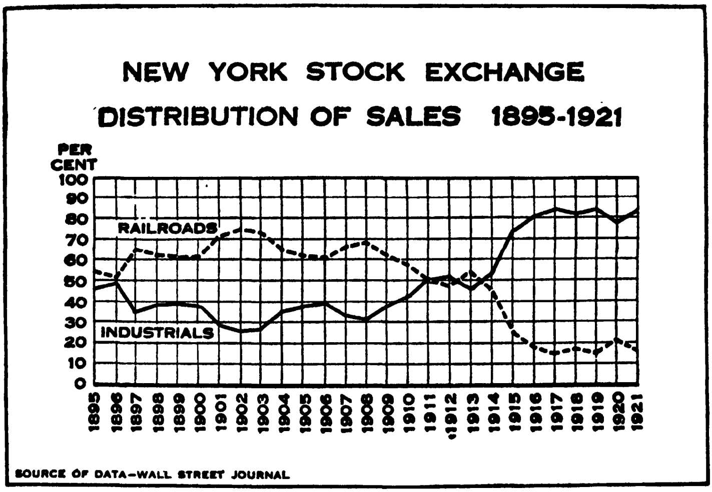


EXHIBIT 120.—A method of showing percentage relations of two components. (Difficult to read.)

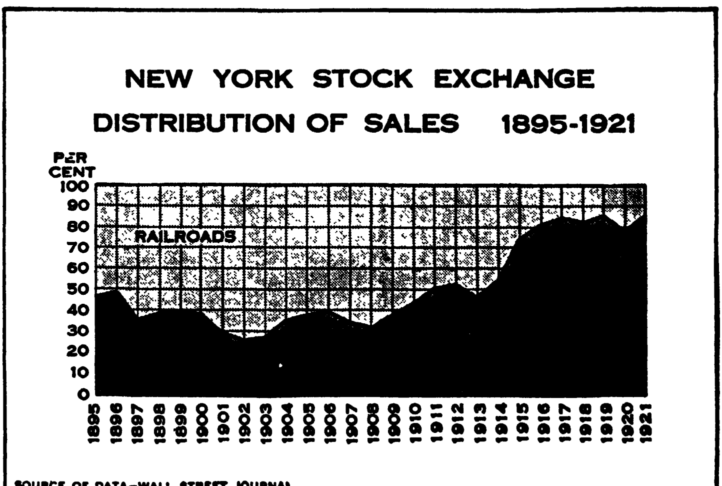


EXHIBIT 121.—A much clearer method of presenting the data of Exhibit 120.

gives the components a sharp contrast and makes the chart clear to many persons who ordinarily would be confused by a curve chart like Exhibit 120.

When several widely varying components are to be shown, it is much easier for the eye to carry them across the period if they are presented in the belt chart form (Exhibit 122) than when they are presented in the bar form as shown in Exhibit 123. However, the method used in Exhibit 123 has the advantage when the emphasis is placed upon comparing the components at certain points, rather than upon showing the changes over the

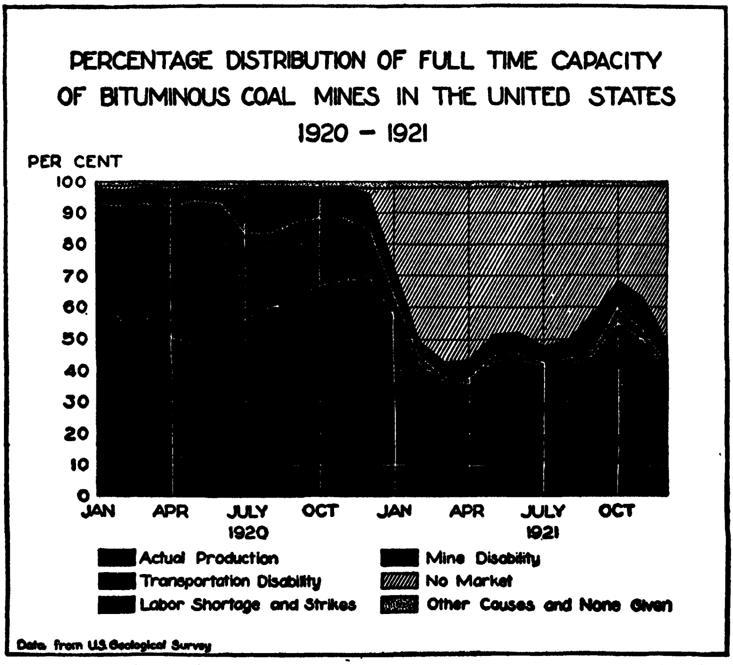


EXHIBIT 122.—Use of component curves to show percentage distribution of several components.

period. The comparisons of the irregular areas in Exhibit 122 are not easy to make, and care must be taken not to compare the widths of the belts. Either the areas or the vertical distances between the curves must be used as the bases for comparisons.

Overlapping—A Common Error.—When the areas between curves are shaded, they must not be assumed to overlap. Such shading (Exhibit 124) is misleading. The chart in Exhibit 124 is wrong, as the lower curve represents an entirely different series from the upper curve and not a component part of the upper curve. Imports are not a part of exports. The curve of exports

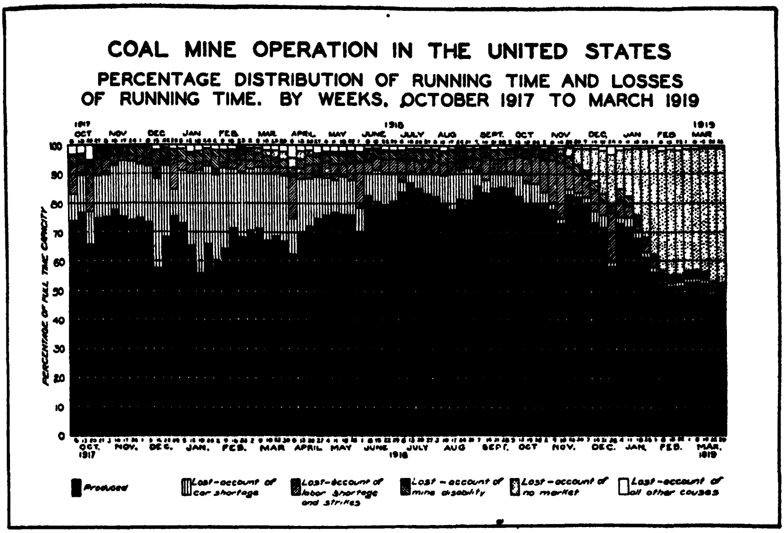


EXHIBIT 123.—Use of component bars to show percentage distribution of several components. (*Courtesy of the United States Geological Survey.*)

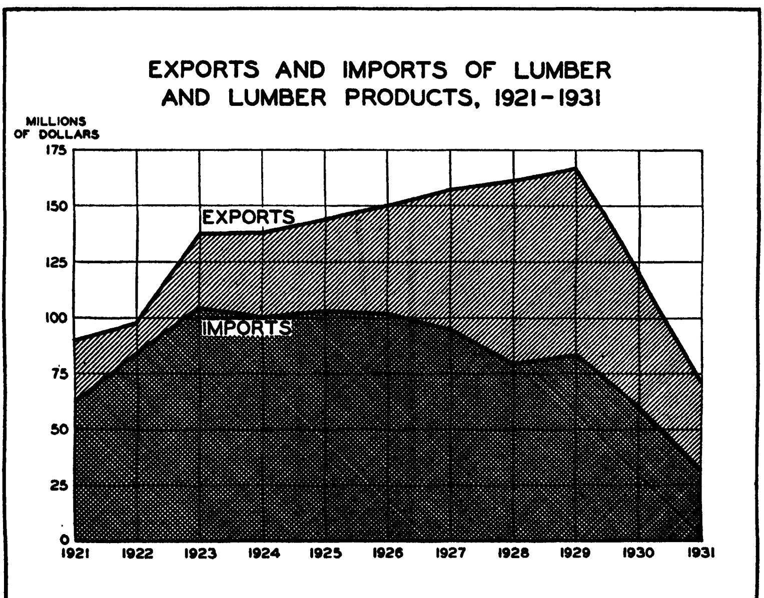


EXHIBIT 124.—A common error in use of shading. Not component parts. Both curves are measured from the zero line.

is measured from the zero line and not from imports as a base, and hence the areas should not be shaded.

Shading Components.—The shading for the different divisions of a component time chart must be chosen very carefully if the graph is to be easy to read. Colors can be used very effectively in component charts and, when used, should be representative, so far as possible, of what is shown. For instance, the original of Exhibit 122 was a colored wall chart on which black represented coal production, and blue represented business depression or no market. This point, however, in the use of colors is only of minor importance and cannot be carried very far. Black and white shading usually is arranged in order from dark to light, but sometimes this order is changed to secure a greater contrast between the different components.

CHAPTER V

HISTORICAL TIME SERIES (*Continued*)

LOGARITHMIC CHARTS

So far, this study of time series charts has included only variations in *size* over the period, and the charts were constructed on simple arithmetic scales. It is the purpose in this chapter to describe the construction and use of charts showing *rates* of change. In this connection a knowledge of the logarithmic scale is necessary. The purposes of arithmetic curves and logarithmic curves are entirely distinct. One cannot be used instead of the other and, in fact, a clear understanding of the arithmetic curve is practically impossible unless one understands also the logarithmic curve. Although in the past there has been considerable prejudice on the part of the business man against the logarithmic chart, he is now using it much more widely since he appreciates its importance and recognizes its simplicity.

RATES OF CHANGE IN A TIME VARIABLE

The preceding chapter covered principally problems of changes in *size* in time variables as illustrated, for instance, in Part *A* of Exhibit 125, in which the size of the population for any one point of time can be compared with the size for any other date. That is, we can see that the population of 1930, for instance, is about twice that of 1890. We cannot, however, in Part *A*, compare the rate of increase of one section of the curve with that of another. Although the curve is steeper from 1920 to 1930 than it is from 1820 to 1830, it does not mean that population is increasing more rapidly in the later period. It means simply that the actual difference in *size* is greater. If the *rate of change* is to be shown, the logarithmic[1] vertical scale can

[1] The Briggs or Common system of logarithms, which is the system in ordinary use, expresses any given number as a certain power of 10. That is, the logarithm of a given number indicates what power of 10 that number is. Thus the logarithm of 10 is 1; the logarithm of 100 or 10^2 is 2; the loga-

be used, as in Part *B* of Exhibit 125, in which the slope of the curve indicates the *rate of growth.* It is readily seen that this *rate* is falling off slightly since the curve is less steep in the later part of the period.

Simplicity of the Logarithmic Curve.—It must be clearly understood at the outset that the two parts of the chart in Exhibit 125 have two different and distinct purposes. Part *A* shows *differences in size* but does not show the *rates of change;* Part *B* shows the *rates of change* but does not show the *differences in size.* Both charts are equally simple, but, because this fact is not fully realized, the arithmetic chart (*A*) has been used more widely in

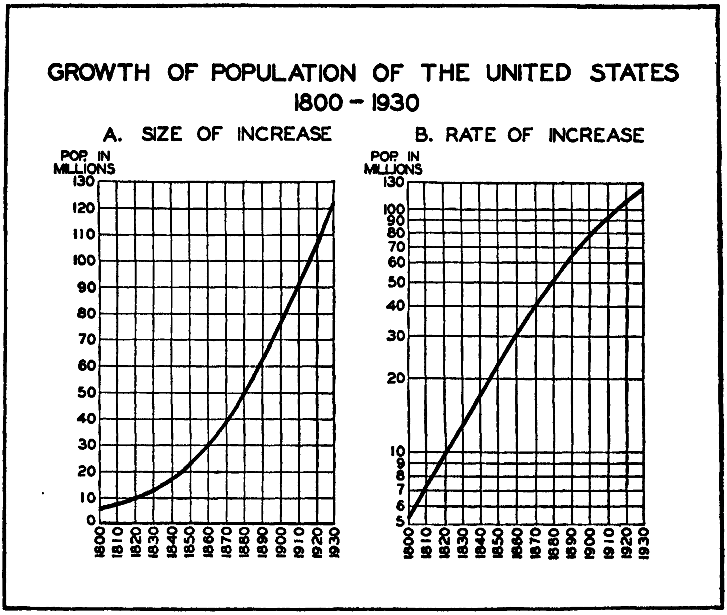


EXHIBIT 125.—Arithmetic and logarithmic curves.

rithm of 1,000 or 10^3 is 3, and so on. If the number is not an even power of 10, the logarithm is computed in the form of a fractional exponent, and thus the logarithms of any assignable number may be approximately expressed. The purpose of logarithms is to facilitate and shorten calculations involving multiplication, division, and the finding of roots and powers. (For a further elementary explanation of logarithms, see John R. Riggleman and Ira N. Frisbee, "Business Statistics," Appendix VI, "How to Use Logarithms," McGraw-Hill Book Company, Inc., New York, 1932.)

charting business statistics than the logarithmic chart (*B*). Most business men regard anything that has to do with logarithms as something highly technical and extremely complicated, but, as a matter of fact, in order to make and use these charts it is quite unnecessary to have even a slight concept of logarithms. In fact, it is sometimes easier to explain the uses of these charts to one who has never studied logarithms than it is to explain them to one who has taken elementary courses which involve the use of logarithms to a limited extent.

Scales for Logarithmic Curves.—Referring again to Part *B* of Exhibit 125, it will be seen that the horizontal scale is ruled arithmetically and that only the vertical scale is ruled logarithmically. Curves made in this combination are commonly referred to by business men as "logarithmic curves."[1] The arithmetic horizontal scale provides uniform spacing for the periods of time, such as months or years. The vertical logarithmic scale, which is in geometrical progression,[2] gives the true relative comparisons of the changes.

On the arithmetic chart, equal intervals on the vertical scale represent equal *differences*. Thus, in Part *A* of Exhibit 126 each vertical spacing represents 200 persons and the curve shows the actual differences for each 10 years. In the logarithmic chart equal intervals on the vertical scale represent equal ratios. Thus, in Part *B* of Exhibit 126, each vertical spacing represents twice

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[1] In business statistics, this chart is commonly called a "logarithmic chart" or a "logarithmic curve." Some writers suggest that it should be called a "semilogarithmic chart," since only one scale is logarithmic, leaving the term "logarithmic" to apply to a chart in which both scales are logarithmic. Two logarithmic scales are so seldom used on a statistical chart, however, that the business user prefers to attach the modifier to this chart calling it a "double logarithmic chart," and to apply the less awkward term "logarithmic" to the single logarithmic scale chart which is in common business use. The terms "semilogarithmic" and "arithlog" are used to describe some of the paper that is on the market for making these charts. Another term which is sometimes used is "ratio chart." (See Professor Irving Fisher, "The Ratio Chart for Plotting Statistics," *Quarterly Publications of the American Statistical Association*, June, 1917.)

[2] A geometric progression is a series of numbers in which every successive term is multiplied or divided by a constant factor, thus 1, 2, 4, 8, 16, 32, etc., or 16, 8, 4, 2, etc. An arithmetic progression is a series of numbers in which every successive term is increased or decreased by a constant amount, as 1, 2, 3, 4, etc., or 2, 4, 6, 8, etc., or 15, 12, 9, 6, etc.

the preceding spacing, or, in other words, each twofold or 100 per cent increase has the same spacing. With reference to the table at the top of Exhibit 126, the increase from 1880 to 1890 represents a difference of 100 persons, and the increase from 1910 to 1920 a difference of 800 persons; these are the facts that are shown by Part *A* (Exhibit 126). However, the rate of increase is the same if 100 increases to 200 and 800 increases to 1,600; hence there is no change in the slope of the curve in Part *B* of Exhibit 126 which shows these *rates* of change.

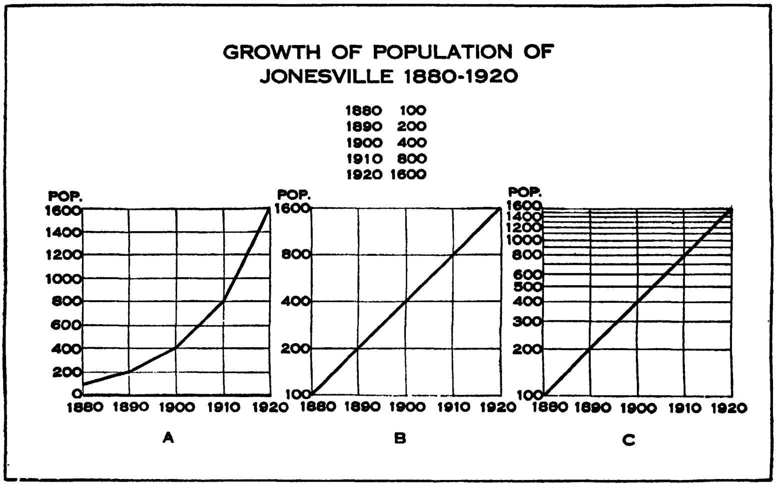

EXHIBIT 126.—Scales in arithmetic and geometric progression.

A logarithmic chart like Part *B* of Exhibit 126 would not be convenient for plotting between the ruled lines, because there are such large intervals of value represented in the upper parts of the scale. A chart is much easier to plot and read if constructed on a basis of tens. Part *C* of Exhibit 126 is the same as Part *B* except that additional horizontal lines have been interpolated at their proper places. In ascending order the lines are spaced closer together, for, if there is to be a line for each 100, there will be none between 100 and 200 (Exhibit 126 *C*); one between 200 and 400; three between 400 and 800; and so on. Note also that these interpolated lines are not equally spaced but are always nearer the line above than the one below.[1]

[1] The divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms. That is, the scale is divided logarith-

Semilogarithmic Paper and Ruling.—When it is desired to use the logarithmic curve, it may be made on special coordinate paper,[1] which is ruled as in Exhibit 127, or the scale may be taken from the statistician's scale[2] or from the slide rule.[3] One complete cycle or deck on a logarithmic scale is usually numbered from 1 to 10, as shown in Exhibits 127 and 128, and the decimal

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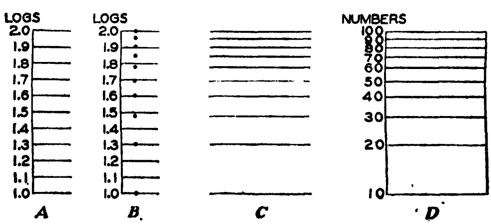
mically, and the designations are not the logarithms but the numbers corresponding to the logarithms. Thus, for instance, 4 is over halfway between 1 and 10 because the logarithm of 4 is 0.6021, which is more than half as large as the logarithm of 10 which is 1; but instead of using the logarithm 0.6021 in the scale, the corresponding number 4 is used.

[1] Semilogarithmic paper, for plotting statistical charts, may be obtained from the Educational Exhibition Co., Providence, R. I.; Keuffel and Esser Co., New York City; and Codex Book Co., New York City.

[2] See page 222 for a description of a statistician's scale.

[3] If semilogarithmic paper or a logarithmic scale is not at hand, a ratio chart may be made by reducing the data to logarithmic terms, and then, using the ordinary arithmetic scale and paper, the logarithms of the given quantities will be plotted instead of the quantities themselves (Exhibit 142, Part *G*, on page 126). If, however, no logarithmic scale is at hand, and it is desired to make a chart on a semilogarithmic coordinate ruling, the logarithmic scale may be laid out as follows: Suppose that it is desired to lay out a logarithmic scale in round numbers from 10 to 100 (10, 20, 30, 40, 50, 60, 70, 80, 90, 100). Look up the logarithms of these round numbers, which would be as follows: 10, 1.000; 20, 1.301; 30, 1.477; 40, 1.602; 50, 1.699; 60, 1.778; 70, 1.845; 80, 1.903; 90, 1.954; 100, 2.000. Next, lay out, arithmetically, a scale on which to plot these logarithms as in *A* as illustrated below. Then plot the foregoing logarithms. This plotting is shown by the points in *B*. Next, rule lines spaced according to the points plotted in *B*, as in *C*.



Then number the lines according to the natural numbers as in *D*, not according to the logarithms. As previously explained in footnote 1 on page 110, the divisions of a logarithmic scale are proportional, not to the numbers themselves, but to their logarithms; that is, the scale is divided logarithmically and the designations are not the logarithms, but the natural numbers.

point may be placed so that the scale will be suitable for the particular curve to be plotted. That is, a cycle or deck may be 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100, 100 to 1,000, etc. Exhibits 128 and 129 show examples of two scales. In Exhibit 129 the

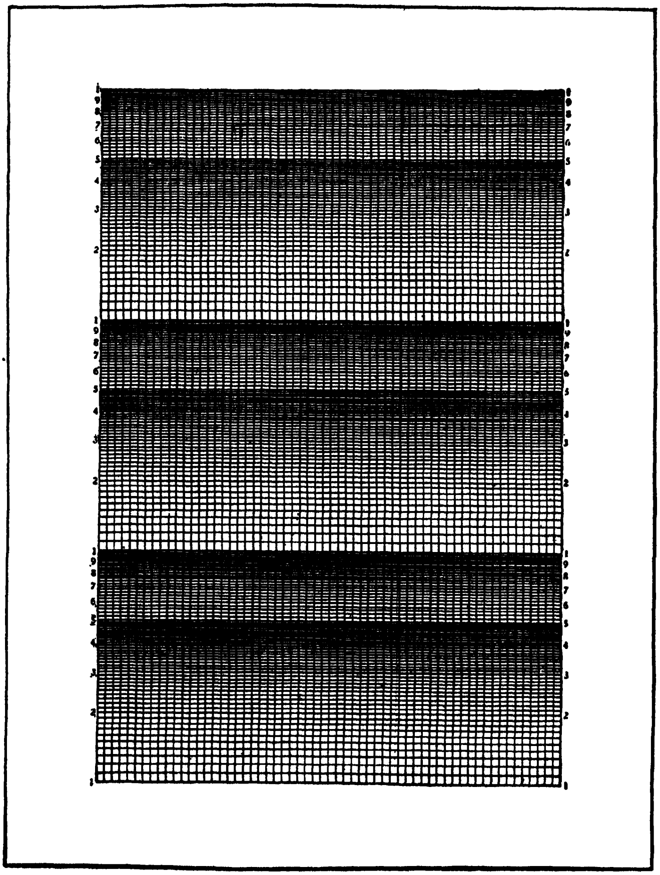


EXHIBIT 127.—Letter size logarithmic paper. (Reduced 2½ times linear.) (Sometimes called semilogarithmic since only the vertical scale is logarithmic.)

scale is continuous through five decks. In all cases each deck is ten times as great as the one immediately below. Thus, Exhibit 128 shows two complete decks numbered (in millions) from 1 to 10 and from 10 to 100, or a continuous range from 1 to 100 millions. Exhibit 129 shows five complete decks, but only the bottom line of each deck is numbered because of the

limited space for lettering. In this exhibit each deck is ten times the one below and a continuous scale (in thousands of pounds) of from 10 to 1,000,000 is given. There are as many lines in a deck in Exhibit 129 as there are in Exhibit 128. In Exhibit 127 extra lines are ruled in for convenience in plotting, but no more

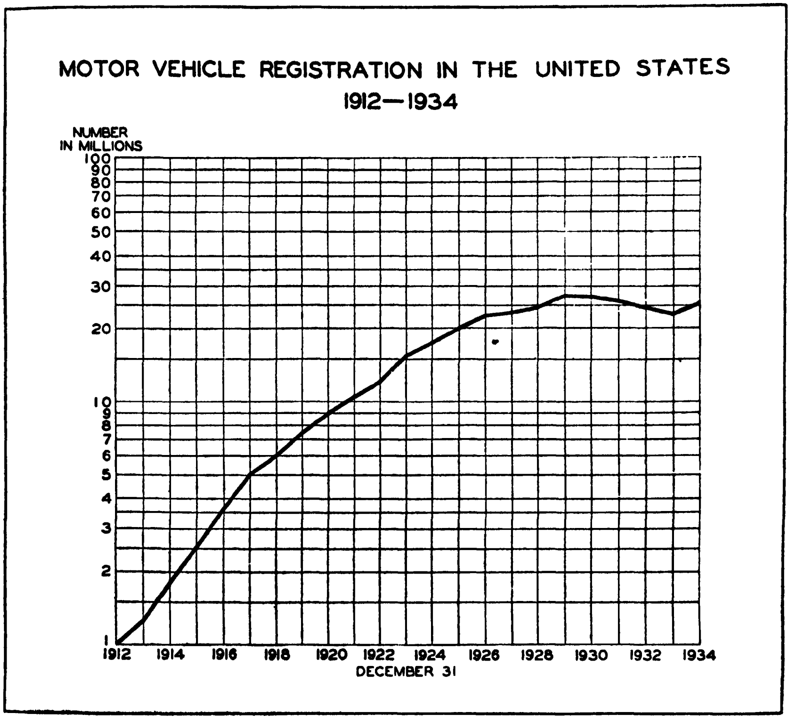


EXHIBIT 128.—Logarithmic chart of two decks. The logarithmic scale is usually designated from 1 to 10 (as above) with the necessary ciphers (or decimal points). The slope of the curve indicates rate of change.

are necessary for reading than are given in Exhibits 128 and 129. Referring again to Exhibit 127, it will be noted that the lines are closer together as the scale ascends, and that, when the point is reached where they would be too close together, some must be omitted.

Test for the Logarithmic Scale.—A simple test that may be applied to the logarithmic scale to determine whether or not it is laid out correctly is to take the scale, dividers, or an edge of paper and see if the spaces from numbers to their doubles are the

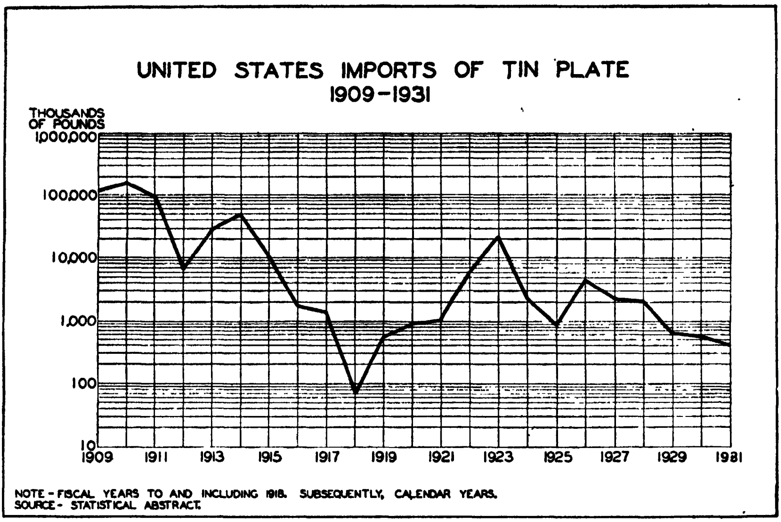


EXHIBIT 129.—Logarithmic chart of five decks. (Space permits vertical scale designations for only the bottom line of each deck. Compare with Exhibit 128.)

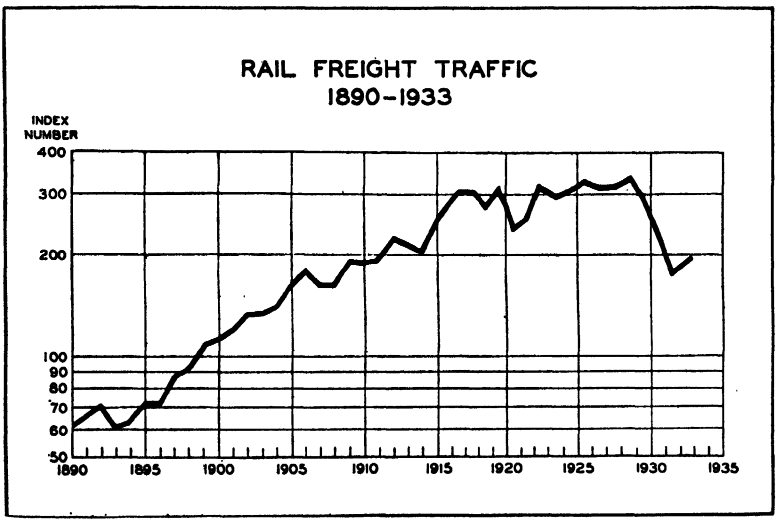


EXHIBIT 130.—There can be no base or zero line on a logarithmic chart. On a logarithmic chart, only that part of the scale sufficient to include the range of the curve need be used. (*Adapted from Automobile Manufacturers Association.*)

same. That is, the distance from 1 to 2 should be the same as from 3 to 6, or 500 to 1,000, or 4,000,000 to 8,000,000, etc. (Doubles are mentioned for convenience only. One to three, one to five, one to ten, or any other ratio would do equally well, as equal ratios should have equal spaces in all cases.)

The Logarithmic Scale Has No Base or Zero Line.—Since the same space stands for the same ratio of magnitudes in all parts of a given logarithmic scale, obviously we can approach zero indefinitely but can never reach it. Consequently, there is no

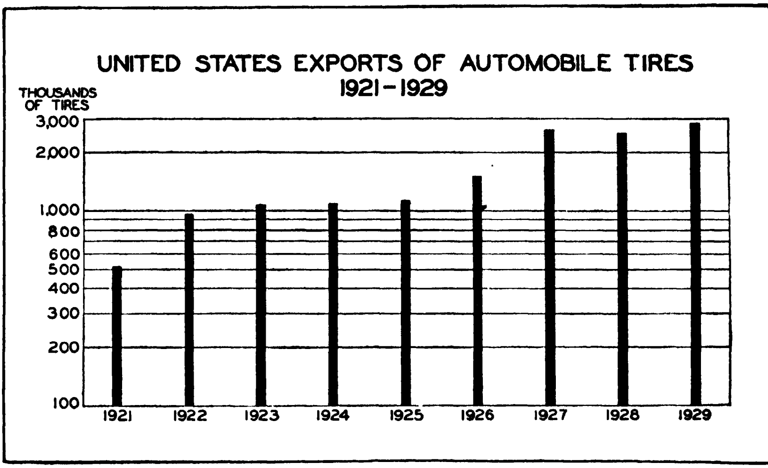


EXHIBIT 131.—Wrong. Such a use of bars on a logarithmic scale is never correct.

zero on a logarithmic scale. Neither is there a base line, for any value on a logarithmic scale is as much base value as any other. The scale may begin with 1, 2, or any other number. It is necessary only to choose a part of the scale sufficiently long to include the data to be plotted, as in Exhibit 130. Since there can be no base, the bottom line of a logarithmic chart should not be emphasized as is the zero line for an arithmetic scale (see Exhibit 130).

Exhibit 131 represents an incorrect use of bars on a logarithmic scale. Since there can be no zero base line on such a chart, the bars cannot represent actual sizes. And such bars have no significance in showing rates of change. A bar chart comparing sizes should never be drawn to anything but an arithmetic scale. The type of chart illustrated in Exhibit 131, however, should not

be confused with that illustrated in Exhibit 136*B* on page 120, which does show relative fluctuations.

Significance of Slopes of Curves.—In the logarithmic chart the index of the rate of change is the slope of the curve. If a curve has a steeper slope in one part than in another, the greater steepness indicates a more rapid rate of change. Equal slopes indicate equal rates of change. Thus, in Exhibit 128 the rate of growth was much more rapid from 1912 to 1923 than from 1923 to 1930. A straight line over a period would represent a constant rate of change. The area under the curve, which is so important on an arithmetic chart, is of no significance on a logarithmic chart.

Smoothing Logarithmic Curves.—The rules for smoothing logarithmic curves are the same as those described for arithmetic curves in the preceding chapter (pages 71 to 74).

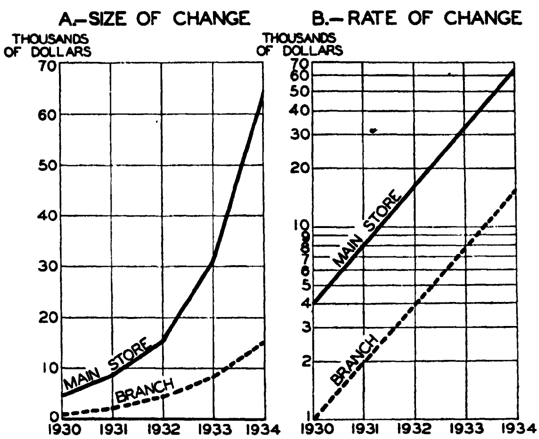
COMPARISONS OF RATES OF CHANGE IN TWO OR MORE VARIABLES

It must be kept in mind that the purposes of the arithmetic curve and the logarithmic curve are distinctly different. The two cannot be combined, and one cannot take the place of the other. This will be brought out more clearly by a consideration of Parts *A* and *B* of Exhibit 132. In *A* we can compare the *size* of one store with the *size* of the other, and it is easy to see that the main store is about four times as large as the branch. We cannot, however, directly compare the *rates* of change in the two curves. That is, we cannot tell which store is growing the faster. To determine this, we can use the logarithmic curve, as illustrated in Part *B* of Exhibit 132, which shows that the rates of change are the same. Of course, one can estimate from the scales what the rate of change is in an arithmetic curve, and what the size difference is in a logarithmic curve, but these estimates are made from values on the scales and not from directly reading the curves.

Business men often attempt to compare the rates of change between arithmetic curves without realizing how greatly they are being misled. For instance, several bankers in a large eastern bank were studying the curves shown in Exhibit 133 from a similar chart of the earlier part of the period which appeared in the *Federal Reserve Bulletin*. After a discussion, they agreed as to the reasons why cotton consumption fluctuates more than wool consumption, when, as a matter of fact, cotton does not

fluctuate relatively so much as wool. This fact is clearly shown by the logarithmic curves in Exhibit 134. If the above-mentioned bankers had appreciated, even in a general way, the use of the logarithmic curve, they would not have erred by trying to read rates, or relative fluctuations, directly from arithmetic curves.

COMPARISON OF SALES OF
MAIN STORE AND BRANCH STORE
1930-1934



	MAIN STORE	BRANCH
1930	$4,000	$1,000
1931	8,000	2,000
1932	16,000	4,000
1933	32,000	8,000
1934	64,000	16,000

EXHIBIT 132.—Comparison of arithmetic and logarithmic curves.

Only Vertical Distances Should Be Compared.—In Part *B* of Exhibit 132 the rates of change are the same in both series. Whenever, in a logarithmic chart, two curves run parallel in the sense that the vertical distance between them remains unaltered, the rates of change are the same. Note that curves which are parallel in this sense may not be separated by equal distances (Exhibit 135). This effect occurs only when curves are far from

straight, but in reading such curves care must be taken to consider the vertical distances between them.

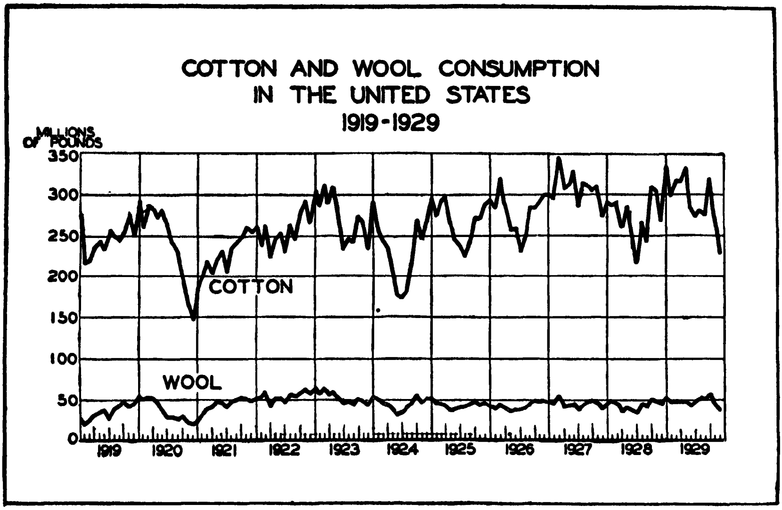


EXHIBIT 133.—Comparing sizes or quantities.

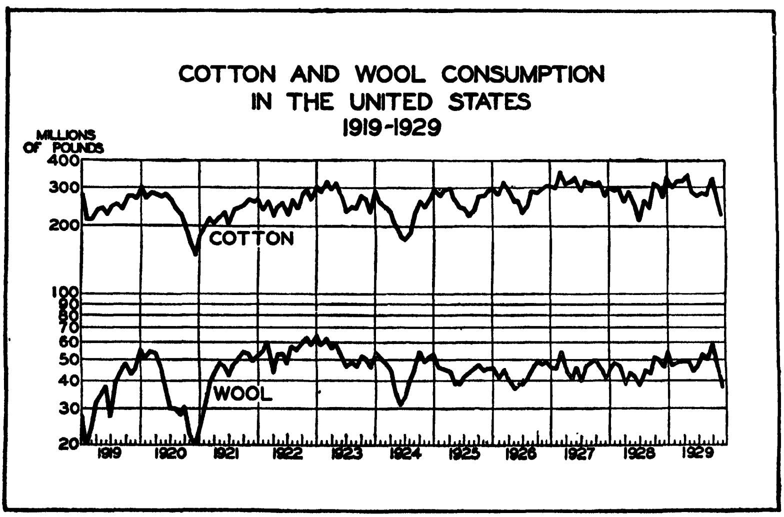


EXHIBIT 134.—Comparing relative fluctuations or rates of change.

Unless the foregoing principle is understood when reading a chart like Exhibit 136*A*, which shows the range of prices proportionally, one may be misled by reading the widths of the

black areas instead of the vertical distances across the areas. This difficulty is overcome in Exhibit 136*B*, in which the length of the bar represents the proportional range. If print cloth varies from 80 cents to $1.20, and cotton varies from 20 to 30 cents, the proportional changes are the same. In Exhibit 136*B*, the distance from 80 cents to $1.20 is the same as from 20 to 30 cents, and, therefore, bars representing these same proportional changes would be of equal lengths.

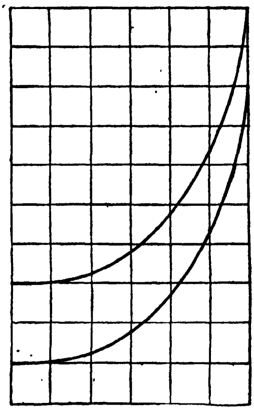


EXHIBIT 135.—These curves are parallel in the sense that the vertical distances between them are the same.

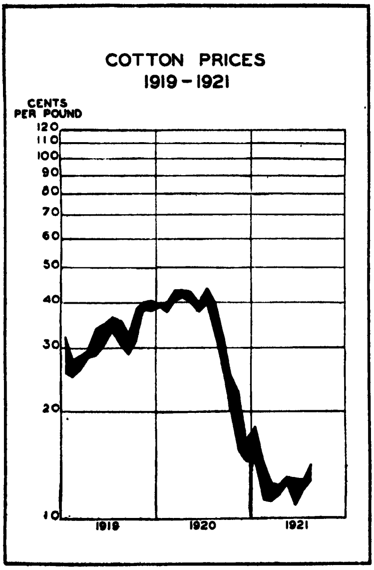
The type of chart illustrated in Exhibit 136*B* must not be confused, however, with the bar chart shown in Exhibit 131 on page 115, which is very misleading and should not be used, as it does not represent relatives, and it cannot represent sizes, since there can be no zero base in a logarithmic scale.

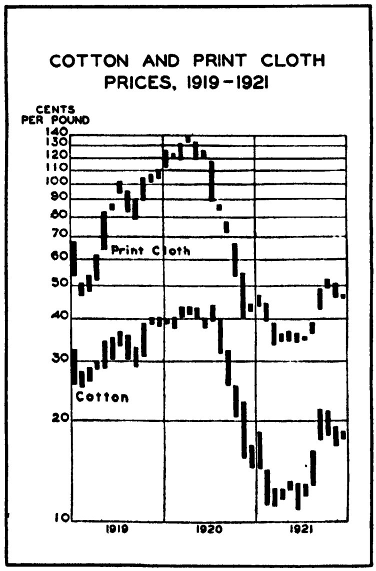
Scale Units and Designations.—In making logarithmic charts, it is desirable to include a scale of the units used; this is an aid in checking and a convenience in reading values from the curve. This scale should be expressed in the same type of units as are used in an arithmetic chart, and for the same data the scales for both kinds of charts would be alike except in spacing. Numbers in the logarithmic scale may have to be omitted as the spacing decreases. The rules for designating units and for positions of scales are the same for both kinds of charts. The rules are the same also for differentiating and designating the different curves. These points have been discussed at length in the preceding chapter.

Common Units Not Necessary in Comparing Rates of Change. Rates of change in different series or in different parts of the same series may be compared when they are not expressed in common units. For instance, in an arithmetic chart we cannot compare the *sizes* of coal and oil productions when coal is expressed in tons and oil in barrels. Different units do not prevent comparing *rates of change* in these series on a logarithmic chart, however, for if oil production increases over the period from, say, two hundred to four hundred millions of barrels, and coal increases

from say, three hundred to six hundred millions of tons, the production has doubled in both instances, and hence the rates of change are the same.

Exhibit 137 illustrates the use of three different scales in such a manner as to bring the curves close enough together to permit easy comparison. As previously mentioned, in connection with Exhibit 99 on page 85, when two or more scales are used in this manner they should be placed on the left-hand side of the graph to avoid the confusion that would result if some were placed on





A *B*

Exhibit 136.—Two methods of showing relative range of fluctuation.

one side and some on the other. The use of different units on a logarithmic chart is further illustrated by Exhibit 138, in which the problem is to compare the four curves with reference to how sensitive they are as indicators of business conditions. Four different designations of the same scale are used (dollars, tons, cents, and shares) in such a manner that the curves are brought close enough together to be compared easily. Not only does this chart show which curve moves first, but it compares the rates at which it moves as compared with the others, and how far it moves relatively. Thus, for instance, stock exchange sales and

Dun and Bradstreet's price index may be compared, not only as to which moves first, but as to which moves the most rapidly and fluctuates the most widely.

Comparing Data of Unlike Time Distribution.—Exhibit 138 presents also the problem of comparing data of unlike time distribution. Thus, stocks of steel sheets are given only for the last day of each month. Dun and Bradstreet's price index is given for the first day of the month, but is based primarily on the

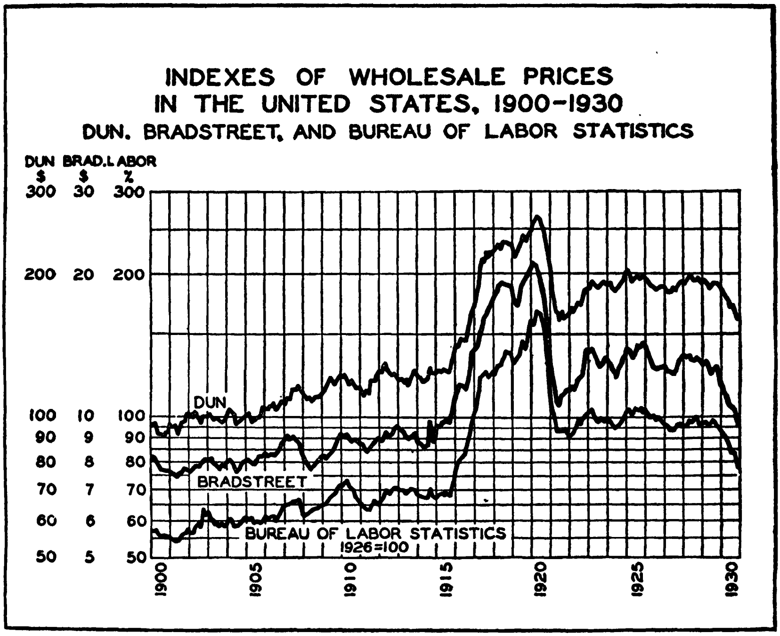


Exhibit 137.—Correct use of different scales on a logarithmic chart.

previous month. New York Stock Exchange transactions are the total number of shares traded during each month, and wheat prices are the average of the high and low for each month. While an exact comparison cannot be made, because the data are unlike, it is often desirable to compare the relative movements of such data, and care must be used in plotting, or an apparent lag may not be a true one. In Exhibit 138 the monthly totals and monthly averages are plotted as of the middle of the month, as the position of least error. Dun and Bradstreet's price index is plotted on the first of the month and stocks of steel sheets

are plotted on the first of the following month; that is, for instance, orders for January 31 are plotted on the February 1 line.

Standardization of Scales.—When it is desired to make logarithmic charts in such a way that they can be compared directly, they should be drawn to the same scales as illustrated in

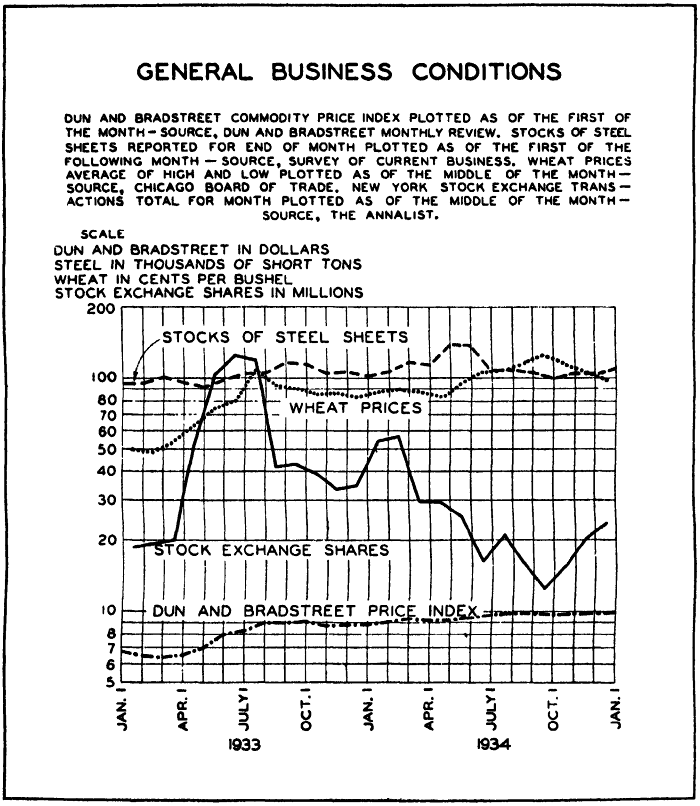


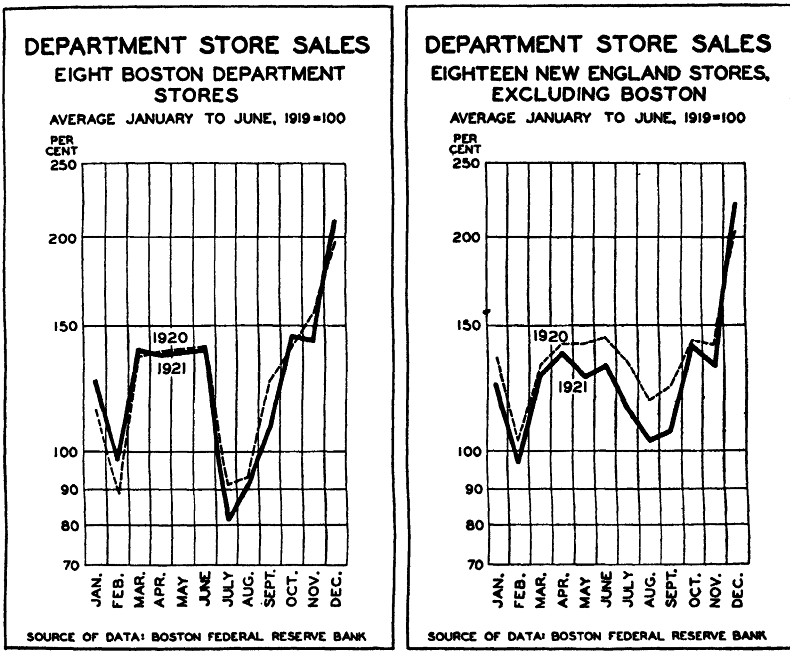
Exhibit 138.—Comparison of unlike time distributions.

Exhibit 139. Then the rates of change of the curves in chart *A* can be compared with the rates of change in chart *B* simply by comparing the slopes. It is easy to see that this could not be done if the intervals for the same ratios in the *vertical* scale were larger in one chart than in the other, or if the *horizontal* scale intervals in one were larger than in the other.

Exhibit 139 also illustrates how the seasonal and other movements of different years may be directly compared. That is,

curves for two different years are plotted to a common scale of months.

Exhibit 140 presents another illustration of the use of uniform scales. In this chart, twenty-four curves are drawn to the same vertical logarithmic scales and the same horizontal arithmetic scales and are, therefore, directly comparable. This exhibit also



A *B*

Exhibit 139.—Comparison of separate logarithmic charts.

illustrates the best method of presenting a large number of curves on one chart.

Shifting Curves for Ease of Comparison.—Since the index of the rate of change is the slope of the curve, the logarithmic chart is sometimes made as in Part *B* of Exhibit 141 to facilitate ease of comparison. In Part *A* of Exhibit 141 the curves are too far apart to show readily the differences in the rates of change, so in *B* these same curves are placed together and the differences can be noted more easily. Both curves still may be read from the scales if the scales are shown as in Exhibit 141*B*. If it is desired to do the original plotting as in Exhibit 141*B*, one scale

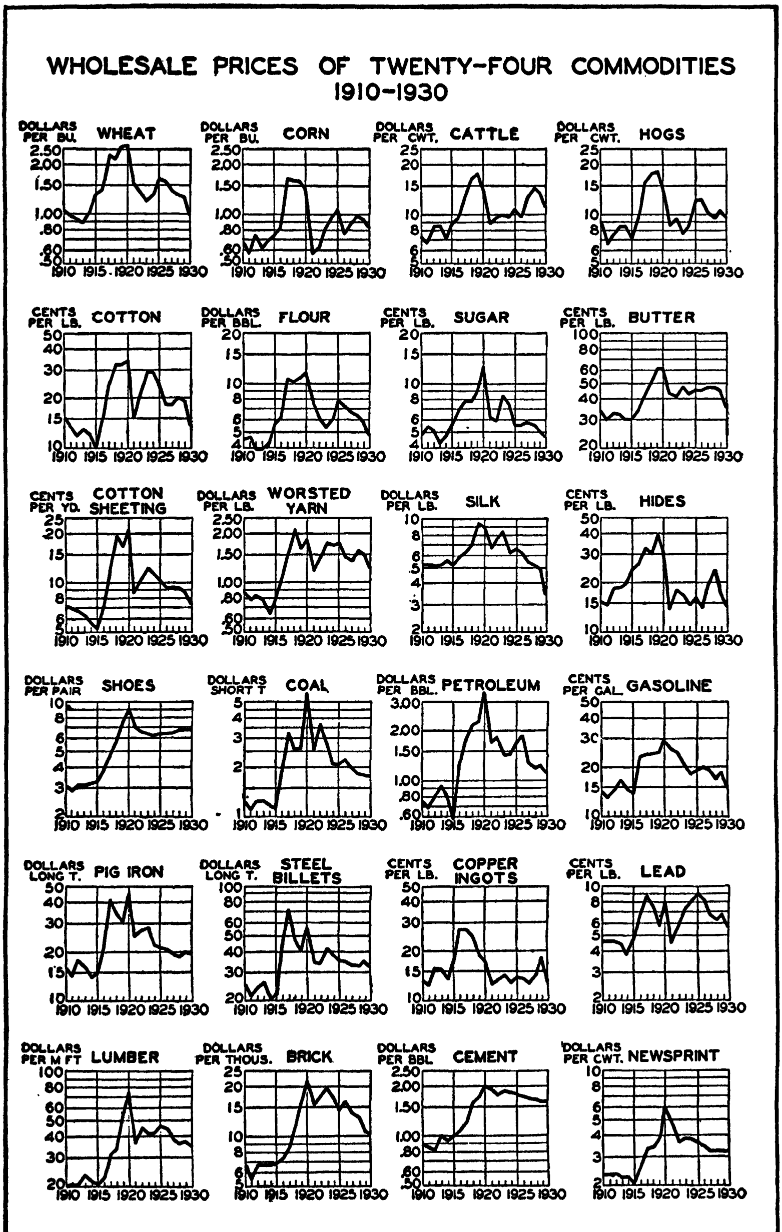


EXHIBIT 140.—Twenty-four curves drawn to the same vertical logarithmic and the same horizontal arithmetic scales.

may be raised or lowered in position along the other by multiplying or dividing it by a constant. Thus, in Exhibit 141*B* it will be noted that every number in the left-hand scale is exactly five times the opposite number in the right-hand scale.

Different charts, when drawn to the same scales, may be superimposed upon each other and shifted about until the curves can be compared easily. Such comparison is greatly facilitated by using a glass copying board such as the one illustrated in Exhibit 248 on page 232.

Percentage Relatives.—Percentage relatives are used when it is desired to compare, not absolute sizes of the quantities or rates

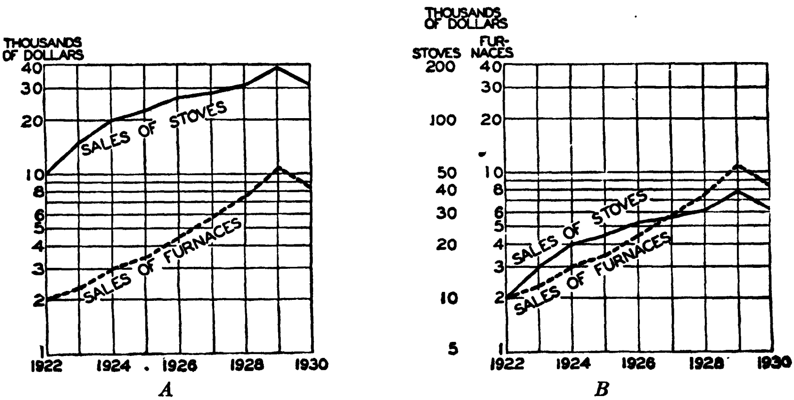


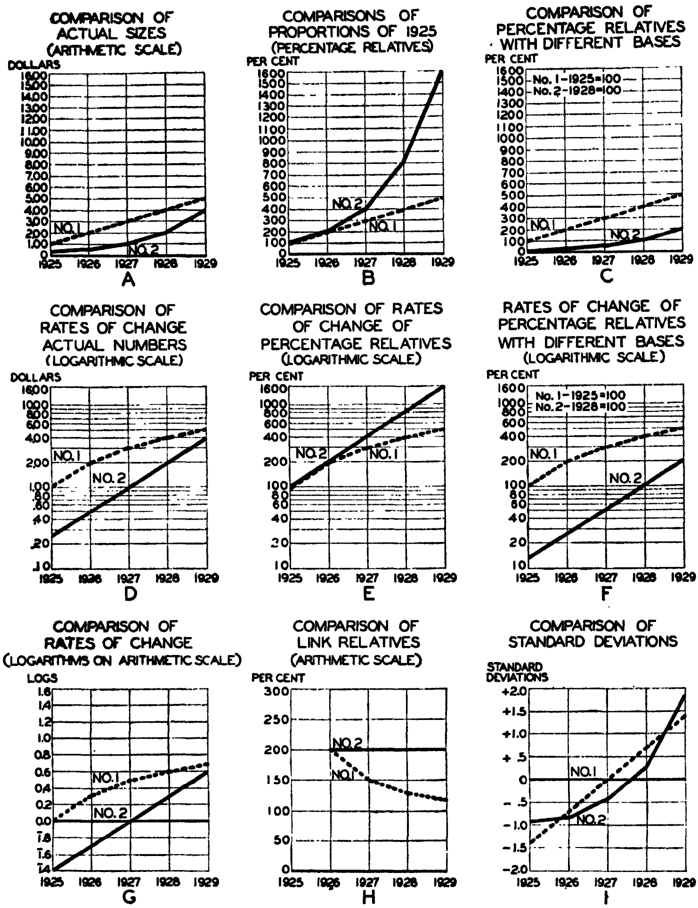
EXHIBIT 141.—Shifting curves to aid comparison.

of change over the period, but the changes that have taken place in terms of percentages of the value at some particular period. In determining percentage relatives, the data for one period or average of a period are taken as the base, usually indicated as 100, upon which the data for the following or preceding periods are computed as percentages.[1] Thus, in Exhibit 142, Graph *A* compares the actual sizes of the two prices over the period, while *B* compares the changes that have taken place since 1925 by showing what percentage of the 1925 value the prices are for the other dates. That is, for 1927 the percentage relative on a base

[1] A rapid and easy method of calculating percentage relatives is to use the slide rule with the following setting:

C scale	Base number (20)	Other number (30)
D scale	100	Percentage relative (150)

PRICES OF STOCKS No. 1 AND No. 2
1925-1929



PRICES OF STOCKS NO. 1 AND NO. 2

YEARS	DOLLARS		PERCENTAGE RELATIVES 1925=100		PERCENTAGE RELATIVES		LOGARITHMS		LINK RELATIVES (%)		STANDARD DEVIATIONS	
	NO. 1	NO. 2	NO. 1	NO. 2	NO. 1 1925=100	NO. 2 1928=100	NO. 1	NO. 2	NO. 1	NO. 2	NO. 1	NO. 2
1925	1.00	0.25	100	100	100	12.5	0.00	$\bar{1}.39$			-1.42	-.95
1926	2.00	0.50	200	200	200	25	0.30	$\bar{1}.69$	200	200	-.71	-.78
1927	3.00	1.00	300	400	300	50	0.47	0.00	150	200	0.00	-.40
1928	4.00	2.00	400	800	400	100	0.60	0.30	133	200	+.71	+.33
1929	5.00	4.00	500	1600	500	200	0.69	0.60	125	200	+1.42	+1.79

EXHIBIT 142.—Various methods of comparing time series.

of 1925 for Stock 2 is 400 (Exhibit 142, Graph *B*), which means that it is 400 per cent of the same stock price for 1925.

The percentage relative chart is often used to compare variables that are expressed originally in different units. That is, for instance, one might compare the percentage changes that have taken place in the production of lumber (board feet) and steel (tons) in terms of some particular date.

The use of the percentage relative chart must not be confused with that of the logarithmic chart. The two are distinctly different. While we can see from Graph *B* in Exhibit 142 that the proportional increase from 1928 to 1929 is greater in No. 2 than in No. 1, we cannot compare the rates of change for the different periods in either curve. The *rates of change* are compared in Exhibit 142 in Graphs *D*, *E*, *F*, and *G*. Although the curve of No. 2 has a constant rate of increase, Graph *B* would give the impression that the rate is increasing; and though the rate is falling off in the curve of No. 1, it appears to be constant in Graph *B*. On a logarithmic scale, the slopes of the curves are the same, whether the actual numbers or the percentage relatives are plotted.

If the percentage relatives of the same base year are plotted as in Graph *E* in Exhibit 142, the curves are automatically shifted so that they are close to each other; this arrangement facilitates the comparison of logarithmic curves, as has been discussed previously. It is important to note that this method is a convenient one for making such a shift. This was the principal reason for reducing the data shown in Exhibit 143 to percentage relatives. A slight spread between the curves in Exhibit 143 is very significant, and unless they are close together it is very difficult for the reader to appreciate the differences in their slopes.

If two or more series of percentage relatives having different base periods are to be studied, they cannot be compared directly on an arithmetic scale without distortion (see Graph *C*, Exhibit 142). Different base periods, however, make no difference in the shapes of the curves plotted on the logarithmic scale; this is illustrated by Graphs *E* and *F* in Exhibit 142, and by Exhibit 144. These different methods of handling the data do not alter the rates of change, and hence there is no difference in what is shown by Graphs *D*, *E*, and *F* of Exhibit 142 or by the three different curves in Exhibit 144. Thus, if one desires to compare the

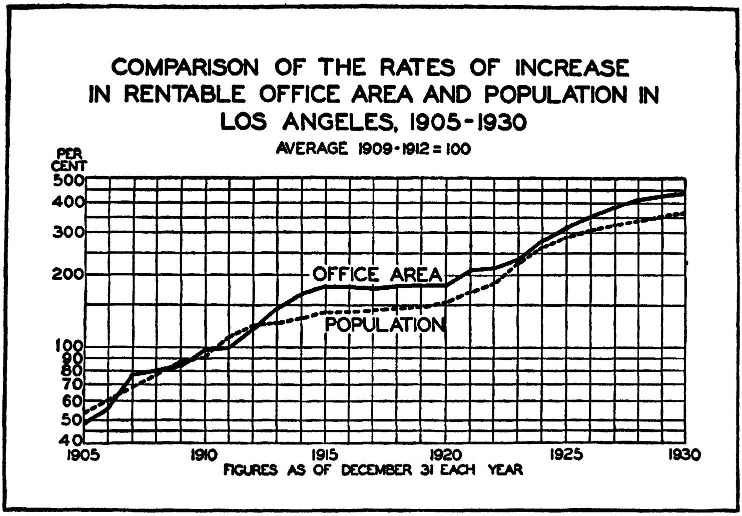


EXHIBIT 143.—Use of percentage relatives for bringing curves together to facilitate comparison.

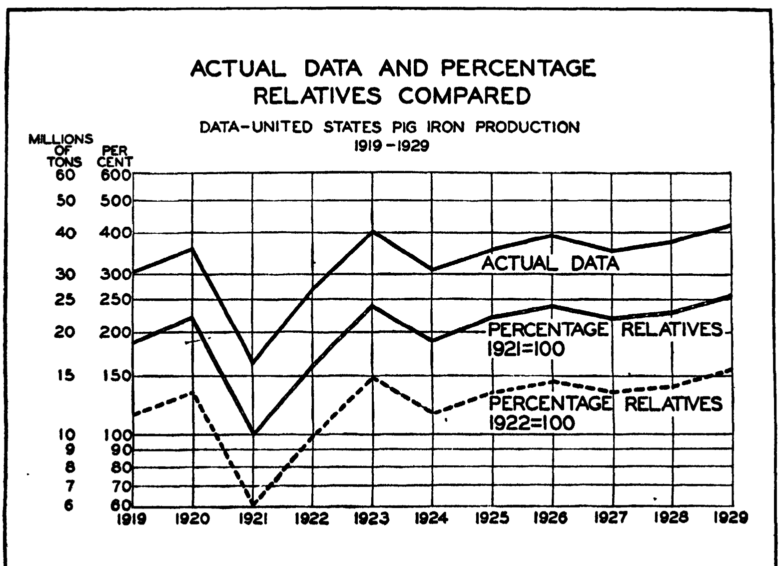


EXHIBIT 144.—Relative fluctuations of actual data and percentage relative series with different bases are the same on a logarithmic chart.

United States Bureau of Labor Statistics Index of Wholesale Prices (1926 = 100) with the Economist (United Kingdom) Index (1913 = 100), he may compare the rates of change by plotting them directly on a logarithmic chart.

If a logarithmic scale is not at hand, a logarithmic chart may be constructed as shown in Graph *G*, Exhibit 142. In this chart the data are reduced to logarithmic terms and then, using the ordinary arithmetic scale, the logarithms of the given quantities are plotted instead of the quantities themselves.[1] Rates of change are shown by the slopes of the resulting curves. It will be noted that the shapes and slopes of the curves in Graph *G* of Exhibit 142 are the same as in Graphs *D*, *E*, and *F*.

Percentage relatives, illustrated graphically by Graphs *B* and *C* of Exhibit 142, should not be confused with link relatives (see Graph *H*). Percentage relatives show all of the values in a series in percentages of one fixed base. Link relatives show each value in a series as a percentage of its preceding value. In Curve 2 of Exhibit 142, Graph *H*, for instance, the value for each year is 200 per cent of the value of each preceding year, and, consequently, this curve is a horizontal straight line. (See also the third column from the end of the table at the bottom of Exhibit 142.)

The Simple Percentage Method.—It is often attempted to show rates of change by another percentage method. And when curves do not greatly diverge, this method gives approximately correct results; but if the curves diverge greatly, comparisons of rates of change are likely to be very misleading. Graphs *C* and *D* of Exhibit 145 illustrate this method, which has been used in an attempt to combine the advantages of both the arithmetic and the logarithmic chart. For the sake of simplicity in illustration, the data shown under "First Assumption" in the table in Exhibit 145 and plotted in Graphs *A*, *B*, *C*, and *D* represent two investments made in 1910, one of $50 and the other of $10 and the rate of increase is the same for both, namely, 25 per cent per annum compound interest. Graph *A* of Exhibit 145 shows the data plotted on an arithmetic scale, while *B* compares the rates of increase. Rates of change can be compared roughly on the

[1] See third footnote on p. 111.

First Assumption

Year	Investment 1	Investment 2
1910	$ 50.00	$10.00
1911	62.50	12.50
1912	78.13	15.63
1913	97.66	19.53
1914	122.07	24.41
1915	152.59	30.52
1916	190.74	38.15
1917	238.42	47.69
1918	298.02	59.61
1919	372.53	74.51
1920	465.66	93.13

Second Assumption

Same as above except:

	Investment 1	Investment 2
1920	Same	$465.66

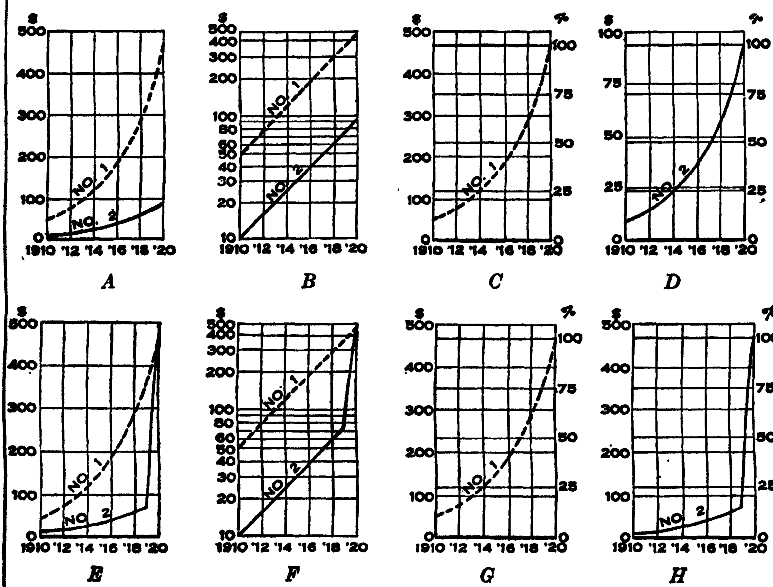


Exhibit 145.—The simple percentage method of juggling scales to show rates of change and example of difficulties.

arithmetic scale between parts of curves that are approximately the same distances from zero, and based upon this principle 100 per cent is given the same space from zero for each curve, as illustrated in Graphs *C* and *D* of Exhibit 145. Thus (in Graph *C* of Exhibit 145), $465.66 equals 100 per cent, while in *D* $93.13 equals 100 per cent, and the intermediate values are located proportionally to these totals. Since both a scale of dollars and a percentage scale are given in *C* and *D*, both values (left-hand scale) and percentages (right-hand scale) can be read from the same chart.

It is the purpose of the makers of charts like *C* and *D* (Exhibit 145) to combine the advantages of *A* and *B*, but as a matter of fact the difference in size, which is the purpose of the arithmetic chart *A*, cannot be compared graphically in *C* and *D*. In regard to the rates of change, we see that in *C* and *D* the curves are alike, but, while we can compare any period in *C* with the same period in *D*, we cannot compare the rates of change between any two periods of the same curve in *C* and *D* as we can in *B*.

The rates of change are shown to be equal for the same periods in *C* and *D* (Exhibit 145), since the curves have the same slopes, but they will be alike only in such an ideal case. Let us assume that the value of Investment 2 increases according to the "Second Assumption" (see table in Exhibit 145) from $74.51 to $465.66 in 1920, instead of to $93.13. Now the arithmetic curves will appear as in Graph *E* of Exhibit 145, and the logarithmic curves will appear as in Graph *F*. The rates of change are the same in both series until 1919. Using the percentage method above described, the curves would appear as in Graphs *G* and *H* (Exhibit 145). This percentage method now fails so far as comparing rates of change is concerned. While the rates of change are the same up until the last year, they are shown by the percentage Graphs *G* and *H* (Exhibit 145) no better than by Graph *E*, which is not intended to show rates of change at all. The figures charted in Graphs *E*, *F*, *G*, and *H* of Exhibit 145 represent an extreme assumption, but no more extreme than the ideal case shown in Graphs *A*, *B*, *C*, and *D*. Additional study of Exhibit 145 will reveal still further the likelihood of error in interpretation when percentage scales are juggled in an attempt to show rates of change.

Standard Deviations.—In a problem such as that of showing business cycles graphically, when the relationship between different series is studied, a situation arises which cannot be solved directly by any of the graphic methods previously discussed.

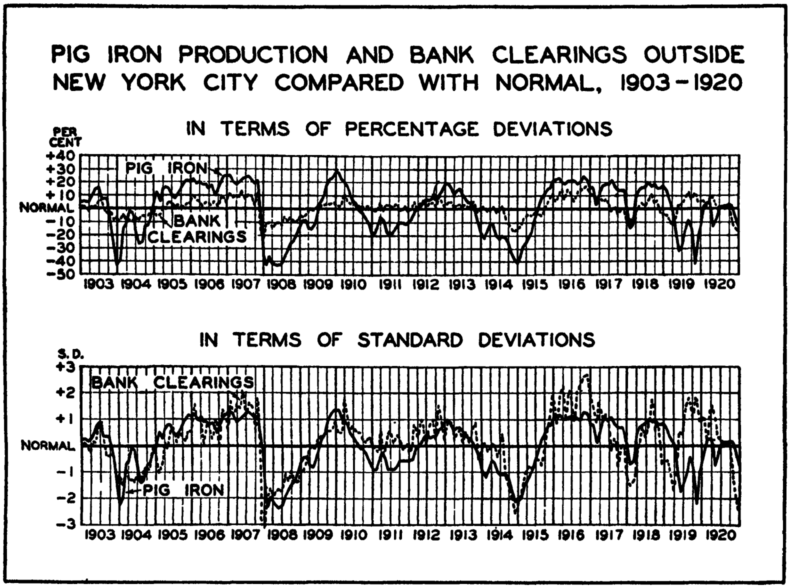


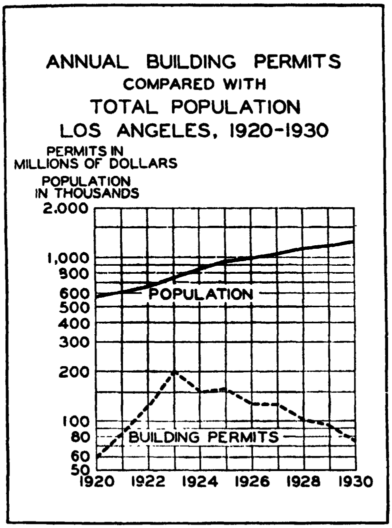
EXHIBIT 146.—Use of standard deviation in graphic presentation. (*Adapted from American Telephone and Telegraph Co.*)

Suppose, for example, that we wish to compare two series, and that in one the percentage deviations range from +17 to −22 per cent, while in the other the range is from +28 to −44 per cent, as shown in the upper graph of Exhibit 146. In such a comparison, a fluctuation of 10 per cent in bank clearings has an entirely different significance from a fluctuation of 10 per cent in pig-iron production. In certain studies of business cycles, it is undesirable to have these differences in amplitude. In such cases, each series may be reduced to terms of its standard deviation, which, for most purposes, is the best method of measuring dispersion in common use at present. The lower graph of Exhibit 146 shows the two series of the upper graph plotted in terms of their respective standard deviations.

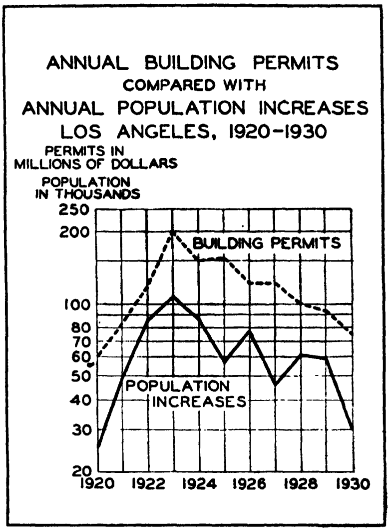
The use of standard deviations is also illustrated in Exhibit 142 in which Graph *I* shows how the series presented in Graphs *A* to

F, inclusive, appear when charted in terms of their standard deviations.[1]

Cumulative and Non-cumulative Series.—As previously pointed out, it is important that the distinction between cumula-



A



B

EXHIBIT 147.—In comparing series (graphically or otherwise), it is important to distinguish between cumulative and non-cumulative data. *A*, Non-cumulative series compared with cumulative series—a difficult comparison to make. *B*, Both series of Part *A* shown on a non-cumulative basis; the comparison is much easier to make.

tive and non-cumulative data be clearly appreciated in the field of graphic presentation. Otherwise, considerable confusion

[1] For the computation of standard deviations, see any standard text on statistics such as G. U. Yule, "Introduction to Statistics," Chap. VIII; W. I. King, "Elements of Statistical Methods," Chaps. XIII and XIV; Horace Secrist, "Introduction to Statistical Methods," Chap. XI; George R. Davies, "Introduction to Economic Statistics," Chap. II; Robert Riegel, "Elements of Business Statistics," Chap. XII; Frederick C. Mills, "Statistical Methods," Chap. V; Harry Jerome, "Statistical Method," Chap. IX; Frederick C. Kent, "Elements of Statistics," Chap. VIII; Robert E. Chaddock, "Principles and Methods of Statistics," Chap. XI; William G. Sutcliffe, "Elementary Statistical Methods," Chap. XV; Edmund E. Day, "Statistical Analysis," Chap. XI; Frederick E. Croxton and Dudley J. Cowden, "Practical Business Statistics," Chap. X; and John R. Riggleman and Ira N. Frisbee, "Business Statistics," Chap. IX.

is liable to result owing to attempts to make direct comparisons of cumulative with non-cumulative series. This is true, whether amounts of change or rates of change are being compared. When a chart similar to that presented in Part *A*, Exhibit 147, was being considered by a certain group of business men, a number of those present raised the question as to why building activity declined so materially while population continued to increase. Though several possible reasons were advanced, the chief difficulty was that the building permit series was non-cumulative, while the population series was of a cumulative nature. As a general rule in such instances, either the annual building permit figures should be compared with the annual increases in population, or the total population figures should be compared with a similar net cumulative series such as total buildings in existence.[1]

The two series of Part *A*, Exhibit 147, are both shown on a non-cumulative basis in Part *B*. That is, the annual building figures are compared with the annual increases in population. It is obvious that the type of comparison illustrated in Part *B* is ordinarily of greater significance and easier to interpret than that of Part *A*.

Summary of Graphic Methods.—In the description of methods as applied to the charts in the last three chapters, certain fundamental rules of practice have been developed. These rules apply to other kinds of charts, and a thorough knowledge of them on the part of the reader will be assumed when other kinds of charts are considered later on in this book. For convenience, both in review and in reference, these fundamental rules have been summarized and presented graphically in Exhibit 148.[2]

Checking List for Curves.—The checking list on page 138 includes the most important points to be observed in making curve charts; it can be used when checking arithmetic and logarithmic time charts, frequency charts, and correlation charts to see whether they are complete.

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[1] It is also often necessary to make other adjustments and corrections, such as eliminating effects of price changes, when such comparisons are made. However, it is the object here only to point out the importance of distinguishing between cumulative and non-cumulative series.

[2] This summary of rules is adapted in part from the "Report of the Joint Committee on Standards for Graphic Presentation," *Quarterly Publications of the American Statistical Association*, Vol. 14, pp. 790–797, 1915.

YEAR	TONS
1900	270,588
1930	555,071

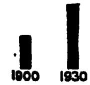


Fig. 1a

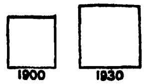


Fig. 1b

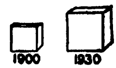


Fig. 1c

1. When possible, linear magnitudes should be used (Fig. 1a) rather than areas (1b) or volumes (1c).

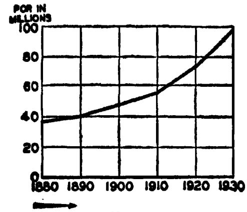


Fig. 2

2. The general arrangement should proceed from left to right (Fig. 2).

3. If the graph is to show sizes directly, the complete scale and zero line should be included (Fig. 3).

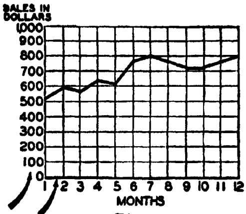


Fig. 3

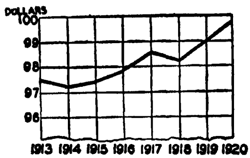


Fig. 4a

4. If the complete scale is not included, the fact should be emphasized by a horizontal break as in Fig. 4a or 4b.

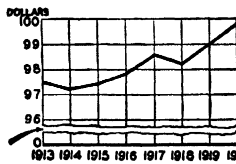


Fig. 4b

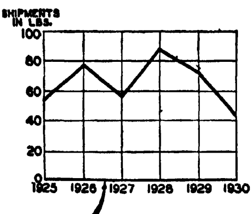


Fig. 5a

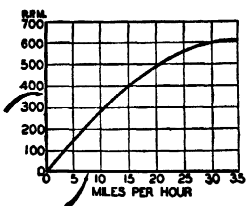


Fig. 5b

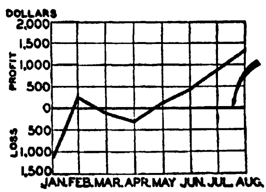


Fig 5c

5. The zero lines of curve scales should be sharply distinguished from the other coordinate lines (Figs. 5a, 5b and 5c). In showing time series, the first vertical line should not be emphasized as a zero line since it does not represent the beginning of time (Fig. 5a).

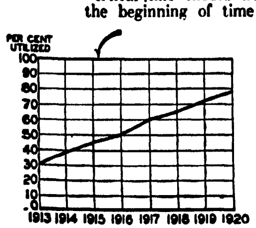


Fig. 6a

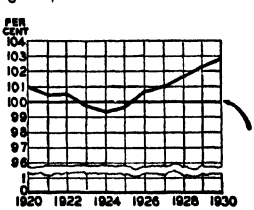


Fig. 6b

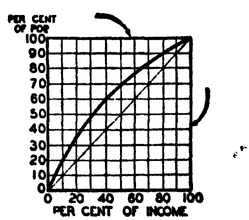


Fig. 6c

6. When the 100 per cent line is used as a basis of comparison, it should be emphasized (Figs. 6a, 6b and 6c).

EXHIBIT 148.—Summary of rules for graphic presentation of business statistics. (For reference and review.) (Continued on pages 136 and 137.)

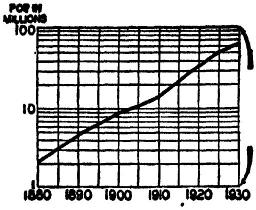


Fig. 7a

7. When a logarithmic chart is made, the top and bottom limiting lines should each be at some power of ten if convenient (Fig. 7a). If not convenient, as when only a small portion of the range is used (Fig. 7b), any other lines may be used as top and bottom limits.

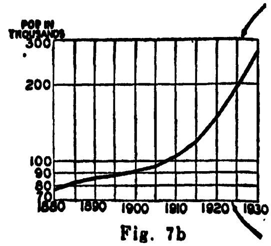


Fig. 7b

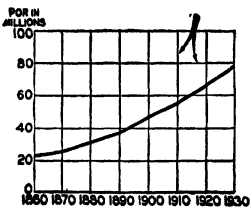


Fig. 8a

8. It is advisable to show no more coordinate lines than are necessary to guide the eye in reading the diagram. Fig. 8a is easier to read than Fig. 8b.

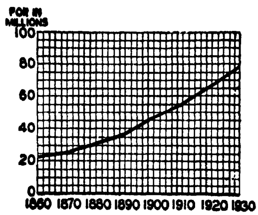


Fig. 8b

9. The curves should be sharply distinguished from the background of coordinate lines (Fig. 9). Since the attention is centered on the curve, it should be made more prominent than the zero line as shown in Fig. 9.

10. When lettering cannot be made to read horizontally as in Fig. 9, it should be made to read from bottom to top as in Fig. 10—never from top to bottom. That is, if it cannot be placed like this: ABC, it should be placed like this: ABC, but never like this: ABC.

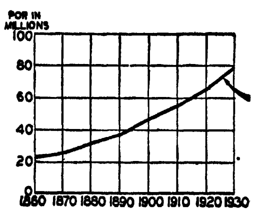


Fig. 9

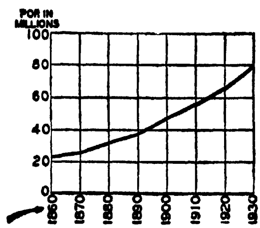


Fig. 10

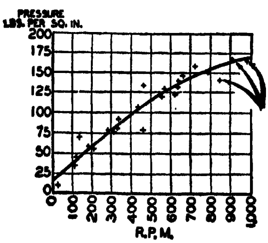


Fig. 11a

11. When irregular or scattered data are plotted, it is often desirable to indicate the points representing the different observations as in Figs. 11a and 11b.

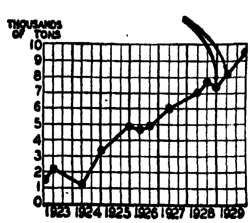


Fig. 11b

EXHIBIT 148 (*Continued*).—Summary of rules for graphic presentation of business statistics.

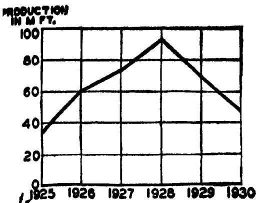


Fig. 12a

12. The scales should be placed at the left and at the bottom of the graph as in Figs. 12a and 12b. If desirable they may be placed at the top and on the right also, but they should never be omitted on the left and on the bottom.

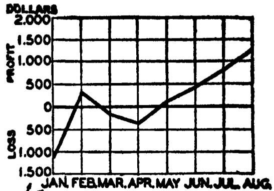


Fig. 12b

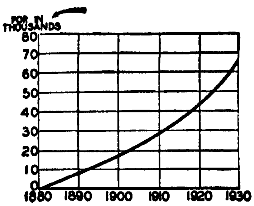


Fig. 13a

13. A caption describing the units used in the vertical scale should be placed at the top of the scale as in Fig. 13a. (Another method is to place the designation along the side but this is less preferable.) When desirable, the horizontal scale should be designated as in Fig. 13b. (Ordinarily it is not necessary to designate years or months.)

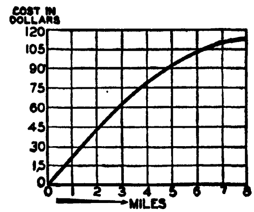


Fig. 13b

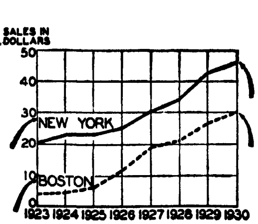


Fig. 14a

14. When two or more curves are presented, they should be differentiated (solid, dashed, dotted lines, etc.) and properly designated (see Figs. 14a and 14b). The method of placing the designations along the curves (14a) is preferred.

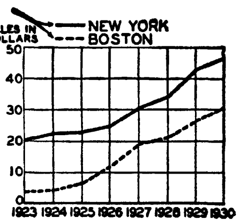


Fig. 14b

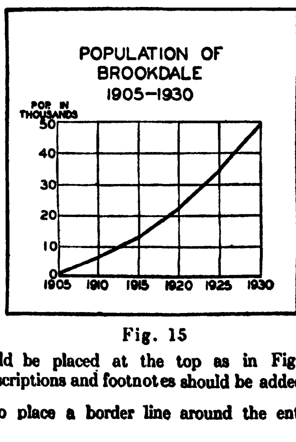


Fig. 15

15a. The title of a chart should be placed at the top as in Fig. 15. It should be as clear and complete as possible. Subtitles, descriptions and footnotes should be added if necessary to insure clearness.

15b. Usually it is desirable to place a border line around the entire chart to give it a finished appearance (Fig. 15).

EXHIBIT 148 (*Concluded*).—**Summary of rules for graphic presentation of business statistics.**

Checking List for Curves

Have you placed your vertical scale (dependent variable) on the left of the graph, and does it read from bottom to top? (If desirable to show the scale on the right, it may be placed there in addition to the scale on the left.)

Have you placed your horizontal scale (independent variable) along the bottom of the graph, and does it read from left to right? (If desirable to place the scale at the top, it may be placed there in addition to the scale at the bottom.)

Have you centered a caption over the vertical scale?

If necessary, have you placed a designation under the horizontal scale?

If you have used two or more curves, have you designated them, and can they be distinguished clearly?

Are your curves heavy?

If you have shaded or colored bars or areas, have you included designations or a key or legend? Are the faces of any such bars free from coordinate or scale lines?

If the vertical scale is arithmetic, is the zero line or 100 per cent base line heavy? (The bottom line of a logarithmic chart should not be emphasized.)

Is your background of coordinate lines light?

Is your title your largest lettering?

Is the title clear and easy to read?

Have you given the reference or source (if desirable)?

Does all lettering read horizontally? (When impossible to letter horizontally, lettering should read from bottom to top—never from top to bottom.)

Are your letters formed according to a commonly accepted style of plain lettering?

Did you check for accuracy?

Have you ruled in a border line?

Have you given your name (lower right) and the date (lower left) if desirable?

Are pencil lines erased?

Will your chart be clear to the reader?

CHAPTER VI

FREQUENCY CHARTS

It is the purpose in this chapter to describe the graphic methods of showing "frequency distributions." Both bars and curves are used to represent frequency distributions, and the forms are similar to those of the time series charts described in the preceding chapters. A frequency distribution chart, as the term implies,

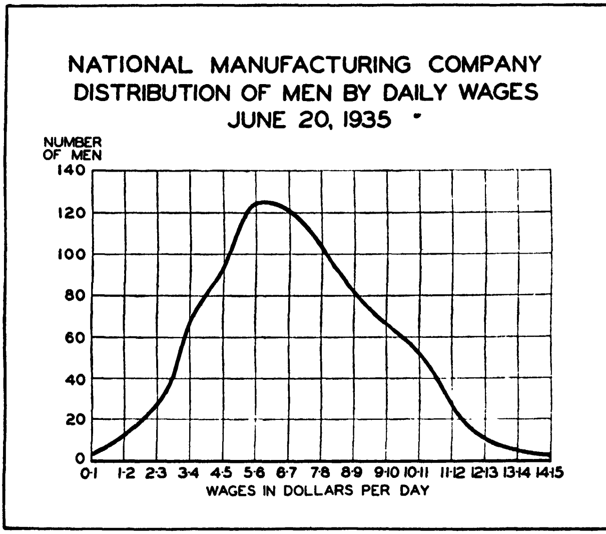


EXHIBIT 149.—A frequency curve.

is a chart which shows the number of occurrences of designated measurements arranged in order. Thus, in Exhibit 149 the vertical scale represents the number of times that each measurement occurs, and the horizontal scale represents the different measurements arranged in order.

Continuous and Broken Series.—In making a frequency chart, the form and treatment depend upon whether the series is continuous or broken. A continuous series is one in which measurements occur at all possible points. A broken series is one in

which the frequencies occur only at certain points which are determined by the nature of the units. Thus, weekly wages of workmen, if expressed in units no smaller than 25 cents, would be a broken series, while their ages would be a continuous series as they might occur at any possible point within the range.

In the field of business the broken series is common. Prices and wages usually are fixed in convenient units. Sales and purchases tend toward round numbers, and not fractions of pieces or fractional weights. Often in this field, series that really are of the continuous type are recorded as broken series. Thus, age will be recorded to the nearest year, weight to the nearest pound, values to the nearest dollar, etc. A broken series may be represented accurately by a bar chart or a broken curve, while a continuous series is most accurately represented by a smoothed curve.

Bars, Broken Curves, and Smoothed Curves in Frequency Graphs.—When a broken series is plotted, the frequency of occurrence of each class is represented by a point plotted directly above the proper class designation. These points may be represented by the tops of bars as in Exhibit 150, or they may be connected by straight lines, making a broken curve as shown in Exhibit 151. In a broken curve, the lines connecting the points have no use other than to aid the eye in comparing their respective heights. There are no values represented on the lines between the points, for there can be no occurrences there.

While the broken curve (Exhibit 151) is the most common method of showing broken series, the bar chart (Exhibit 150) is ordinarily easier for untrained readers to understand. When the significance of the broken curve is likely to be misunderstood, a simple bar chart, in which the frequencies of the different classes and their distribution are indicated simply by lengths of bars, is ordinarily the most satisfactory method of presentation. When bar charts are used to show frequency distributions, vertical bars, rather than horizontal, should be used, so that they may be read in the same direction and manner as curves.

Although in business practice, as a matter of convenience, continuous series are often plotted as broken curves, a smoothed curve is more significant of the distribution of a continuous series than a broken curve or a bar chart. The smoothed curve (Exhibits 149, 153, and 154) should be used when it would be

instrumental in giving a more accurate idea of the distribution than bars or a broken curve. It implies that the distribution is

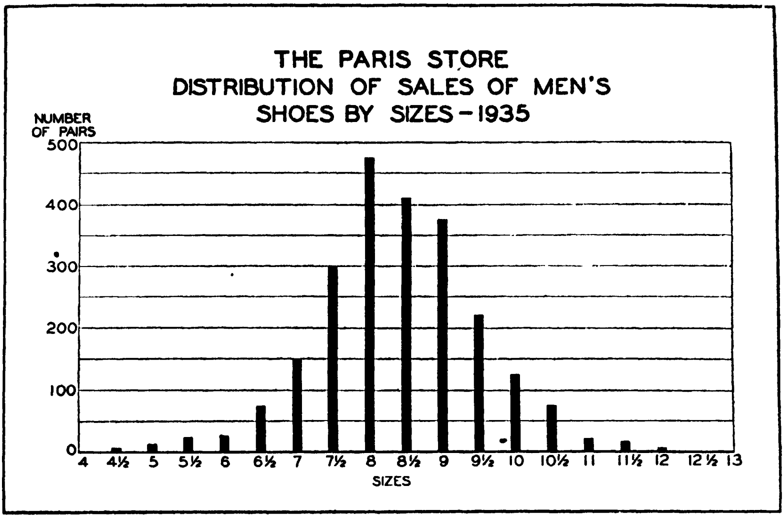


EXHIBIT 150.—A frequency bar chart.

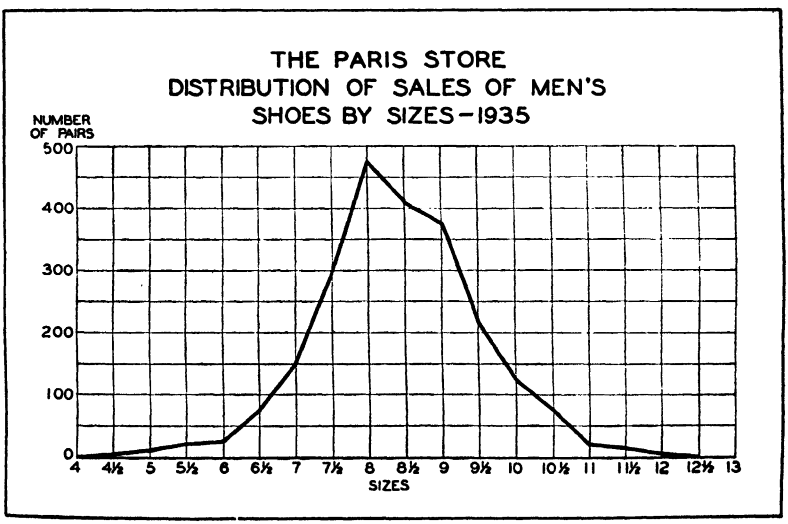


EXHIBIT 151.—A broken frequency curve.

continuous, and that the larger the number of observations the fewer will be the number of irregularities. The distribution of a

smoothed series changes gradually; there is no abrupt change from one class to another. The directions of the changes are indicated by the gradual turns in a smoothed curve. The variations from one extreme measurment to the other are regular and gradual, and the curve should be made free from sharp angles, the shape being influenced at each point by the positions of the adjoining points (Exhibits 153 and 154).

Scales for Frequency Graphs.—Equal distances on each scale of a frequency graph should represent equal values. Both scales should be divided into units of round numbers so that they may be read easily. The positions of scales and their designations are illustrated in Exhibits 151 and 153. That is, they should be outside the coordinate lines—below and to the left. If it is desirable to have scales above the graph and on the right, they may be placed there in addition to the scales underneath and on the left, but in no case should the scales underneath and on the left be omitted.

The vertical scale of a frequency graph should begin with zero, but the use of zero on the horizontal scale is not necessary unless the range of classes includes the interval which has zero as its lower limit.

In these respects the frequency curve is much like the arithmetic time curves previously described. The vertical distances from the zero base to the curve represent the number of occurrences, and, unless the whole scale including zero is used, a wrong impression will be given, as explained in Chapter IV.[1] The horizontal scale shows the measurements represented, and usually the inclusion of zero in this scale, when the lower limit of the range is relatively far from zero, adds nothing and may detract from the value of the graph.

Class Intervals.—In general, the class intervals of a frequency graph should be equal. Otherwise the chart is likely to give a wrong idea of the distribution. Exhibits 152 and 153 show the same data grouped by unequal and equal classes. In Exhibit 152 the range is expressed with open ends, and the first definite class intervals vary from $250, at the lower end of the range, to $1,000 at the upper end. In Exhibit 153 the intervals are uniformly

---

[1] See pp. 62 to 64, inclusive.

$500. Comparison of the two curves indicates clearly the misleading effect of grouping by unequal class intervals.

When cumulative curves are plotted, the general rule of equal class intervals does not prevent the use of smaller intervals for parts of the range over which the frequency varies rapidly, but the scale must be spaced accordingly (Exhibit 163 on page 153). Equal spaces should represent equal values as illustrated.

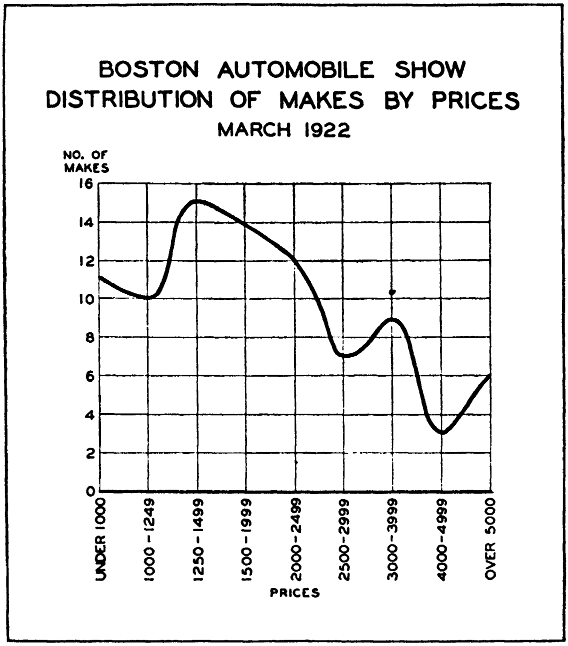


EXHIBIT 152.—Incorrect because of unequal class intervals. (Compare with Exhibit 153.)

Quite often statistical publications use class intervals of unequal magnitudes, and it is difficult, if not impossible, to present the data in the form of a frequency graph. There are, however, occasional exceptions to the rule that class intervals should be equal. It sometimes happens in actual work that class intervals of uniform size are not practical. For instance, in a marketing survey, which made necessary the classification of consumers by income groups, one class interval was $1,750 to $2,000 and another was $4,000 to $5,000. The difficulty here is with the unit. A dollar means more in the first group than in the second. One

dollar probably means as much in the first group as four dollars means in the second group. In relation to the particular commodity under consideration, variations in standards, customs, etc., in the second group were no greater than in the first and, therefore, the purchasing group in the $4,000 to $5,000 class was as uniform as the one in the $1,750 to $2,000 class.

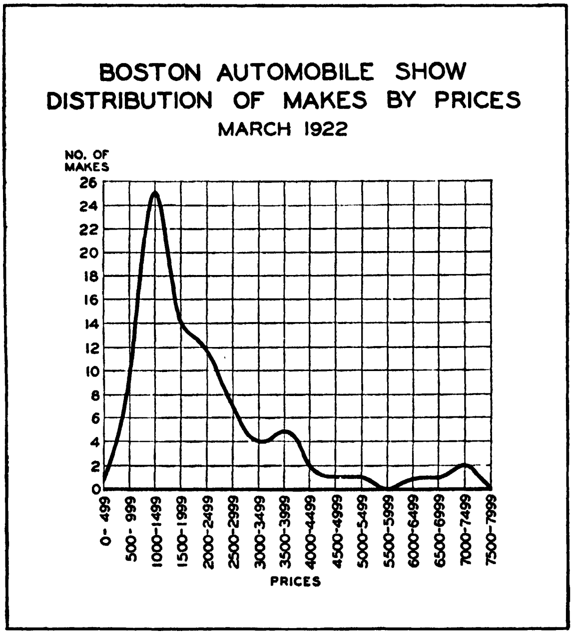


EXHIBIT 153.—Correction of Exhibit 152. Class intervals equal.

The number of class intervals to be shown on a frequency graph depends upon the character of the data. Usually it is best so to choose the interval that the entire number of classes is between fifteen and twenty-five.[1] A number of classes of less than, say, ten leads in general to very appreciable inaccuracy; and a number of over, say, thirty makes an unwieldy graph. This rule, of course, is not inflexible. The classes should be wide enough to make the distribution fairly smooth; that is, vacant classes should not occur, except at the extremes of the range, and yet the classes

[1] YULE, G. U., "An Introduction to the Theory of Statistics," p. 79, Charles Griffin & Company, Ltd., London, 1924.

should not be so wide as to obscure the tendencies of the distribution.

A class interval should not be omitted from a series simply because there are no occurrences in that class. All of the intervals should be included, and the curve or bar brought down to zero when there are no occurrences (see Exhibit 153).

Designating Class Intervals.—The most common method of designating class intervals in the horizontal scale is by expressing

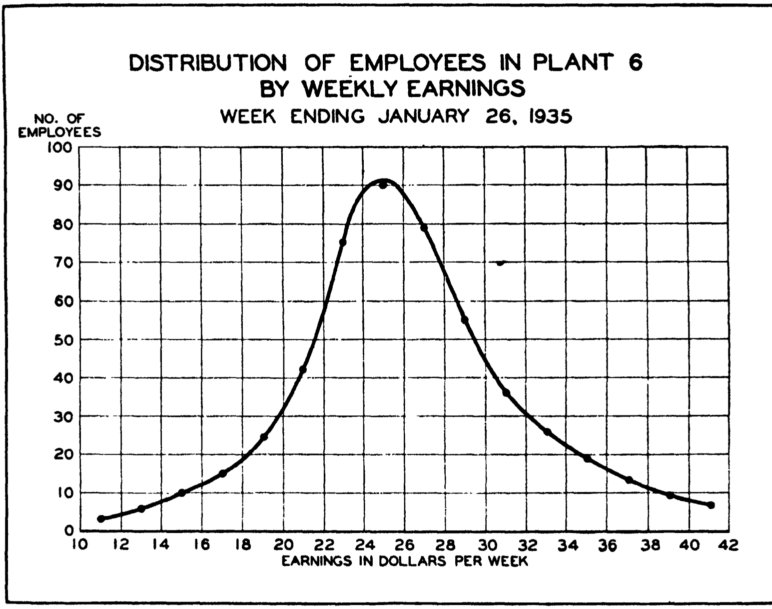


EXHIBIT 154.—Designating class limits by adjoining figures. (Not common practice. Compare with Exhibit 159, which shows this same curve with class limits designated by a more generally used method.)

the lower and upper limits of each class as illustrated in Exhibits 149 and 153. The limits expressed in Exhibit 153 are, of course, inclusive. That is, for instance, the number of prices that falls between 500 and 999, inclusive, will be plotted above this class. Note that, theoretically, the method illustrated is better than the method of expressing the classes as 500 to 1,000, 1,000 to 1,500, 1,500 to 2,000, etc., because in this latter plan it is not understood generally in which group 1,500, for instance, would fall—the 1,000 to 1,500 interval or the 1,500 to 2,000 interval. In practical work, however, the latter method is often used because it is

somewhat more simple and convenient, for instance, to say 1 to 2, 2 to 3, etc., than it is to say 1 to 1.99, 2 to 2.99, etc., and, if the classes are well chosen, any theoretical error is not important enough to interfere with the interpretation or application. Exhibit 149 on page 139 illustrates this method of expressing class limits.

Another method of designating class intervals is illustrated in Exhibit 154. Here any two adjoining figures designate the limits of a class, and the frequency point is plotted above the center of the class so designated. This method, however, may not be so clear to the average reader as the method illustrated in Exhibits 149 and 153.

Occasionally, in plotting business data, the range of variation is so small that there will be no class intervals, and the unit will serve instead. Or, if there is a unit width of class, it is sometimes designated by a single figure as, for instance, $4, which would include all items from $3.50 to $4.49, inclusive, or 10 years, which would include all ages from 9.5 to 10.49 years, inclusive, as illustrated in Exhibits 165 and 166 on page 155.

Smoothing a Frequency Curve.—When a curve is smoothed, care should be taken to keep the area under the curve the same as if a column diagram (Exhibit 155*A*) or a broken curve or frequency polygon (Exhibit 155*B*) were constructed. In the diagram of Exhibit 155*A*, the total area represents the total number of observations, or a certain area represents a certain number of observations. For instance, if two spaces on the horizontal scale represent $10, and two spaces on the vertical represent 100 occurrences, $100 \times 10 = 1{,}000$, or the number of observations represented by an area of four spaces. The total area under the curve in Exhibit 155*B* is the same as that in Exhibit 155*A*. This fact is illustrated by Exhibit 155*C*, which shows that the areas cut off of a column when a broken curve is drawn are the same as the areas added by the curve to the adjoining columns. While the total area is correct in both *A* and *B* of Exhibit 155, the area over any one class is not correct for a continuous series in the frequency polygon of Exhibit 155*B*. For instance, the middle class has lost without gaining anything (Exhibit 155*C*). This must be considered when the curve is smoothed, and usually the best method is to draw the curve so that the areas gained and lost by each column will be equal

(Exhibit 156). Thus, the top of the curve will slightly overtop the highest plotted point. A little thought at this point will indicate that it is reasonable to expect such a distribution within this particular class interval. In actual practice, the column diagram

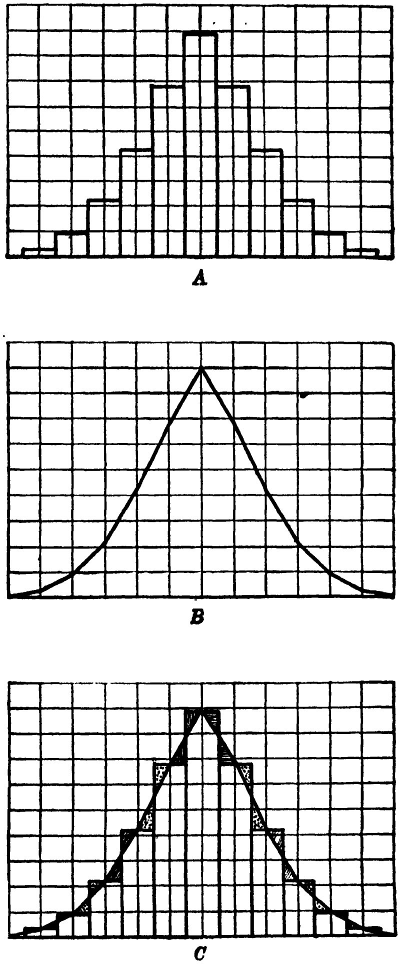


EXHIBIT 155.—The relation between a column diagram and a broken curve.

is not usually made as a basis for drawing the smoothed curve, but the curve is sketched directly from the plotted points, allowances being made by keeping in mind the area distribution as above described. If the series begins and ends with an infinitely small

number of occurrences, the curve should begin and end on the zero line as illustrated in Exhibit 156.

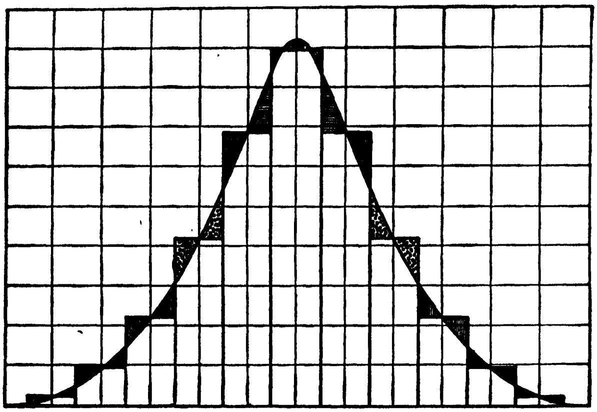


Exhibit 156.—The relation between a column diagram and a smoothed curve.

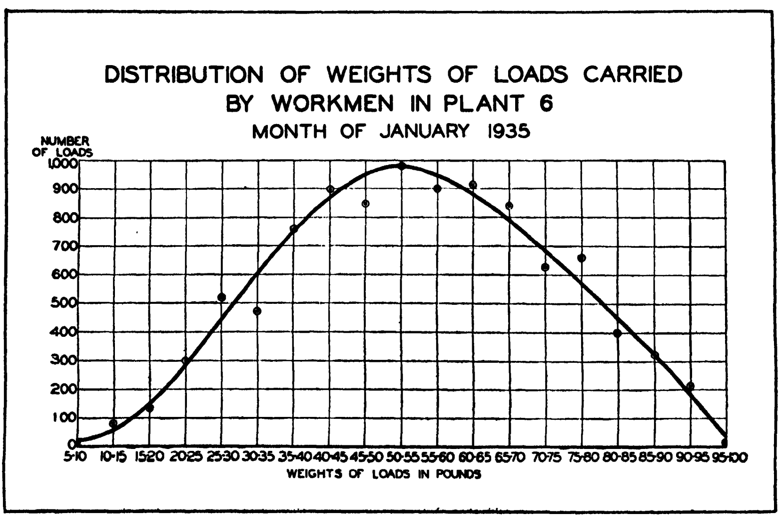


Exhibit 157.—Showing points when curve is freely smoothed.

It must be remembered that business data are full of irregularities, and that smoothing must not obscure any except minor irregularities which are not thought actually to exist.[1] If an

[1] While the "bell-shaped curve" is known as the normal frequency curve (*A*), business data more commonly describe some other form or combination of forms. The fundamental simple types of distributions may be considered as four in number,—namely, the symmetrical (*A*), the moder-

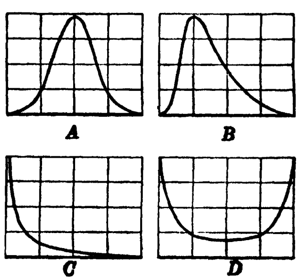
DISTRIBUTION OF EMPLOYEES IN PLANT 6 BY WEEKLY EARNINGS
Week ending January 26, 1935
Cumulative frequencies

Weekly earnings in dollars	Number of employees		
	Simple frequency	Cumulative frequency	
		Less than*	More than*
10.00–11.99	3	3	509
12.00–13.99	6	9	506
14.00–15.99	10	19	500
16.00–17.99	15	34	490
18.00–19.99	24	58	475
20.00–21.99	42	100	451
22.00–23.99	75	175	409
24.00–25.99	90	265	334
26.00–27.99	79	344	244
28.00–29.99	55	399	165
30.00–31.99	36	435	110
32.00–33.99	26	461	74
34.00–35.99	19	480	48
36.00–37.99	13	493	29
38.00–39.99	9	502	16
40.00–41.99	7	509	7

* In this table, "Less than" applies to and includes the upper limit of the class, while "More than" applies to and includes the lower limit.

EXHIBIT 158.—Simple frequency data cumulated on both "less than" and "more than" bases. (See Exhibits 159, 160 and 161, for charts of these data.)

ately asymmetrical (*B*), the J-shaped (*C*), and the rarely occurring U-shaped



(*D*). (See G. U. Yule, "Introduction to the Theory of Statistics," pp. 87–105, Charles Griffin & Company, Ltd., London, 1924.)

irregularity is such that it would not be eliminated by further sampling, it should not be smoothed out. If smoothing is freely indulged in, it is well to indicate the points from which the curve is made (Exhibit 157). Absolute rules, however, cannot be laid down for smoothing frequency curves, for each problem must be considered according to the representative character of the data, and the curve should be plotted which will best suit the purpose for which the graph is made.

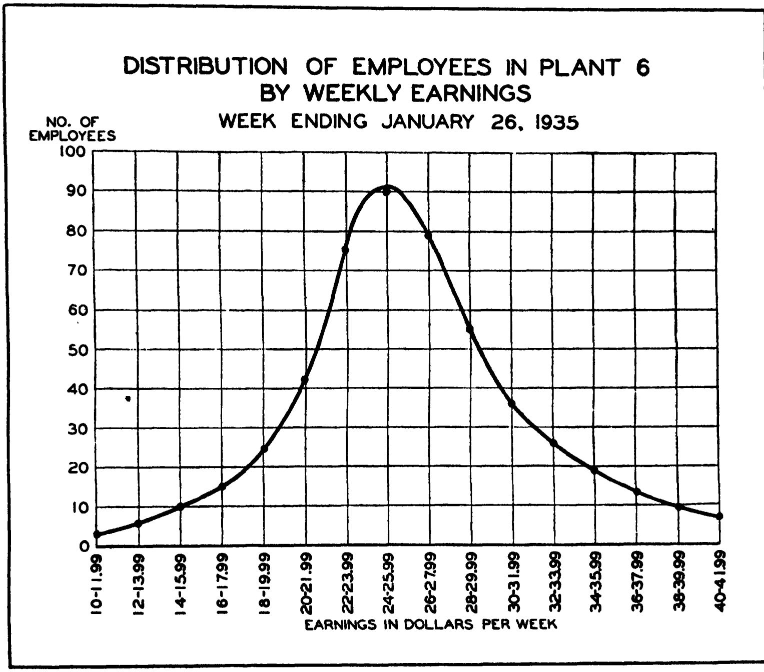


EXHIBIT 159.—Frequency curve of the simple frequency data shown in Exhibit 158. (Compare with Exhibits 160 and 161.)

Cumulative Frequency Curves.—So far, only simple frequency curves have been considered, such as in Exhibit 159, which is plotted from the first column of the table in Exhibit 158. Another form which is widely used in presenting frequency distributions of business data is the cumulative frequency curve.[1] Frequency series are made cumulative when successive frequencies are added

[1] See W. C. Brinton, "Graphic Methods for Presenting Facts," pp. 149–199, McGraw-Hill Book Company, Inc., New York, 1914, for further examples of the use of this curve.

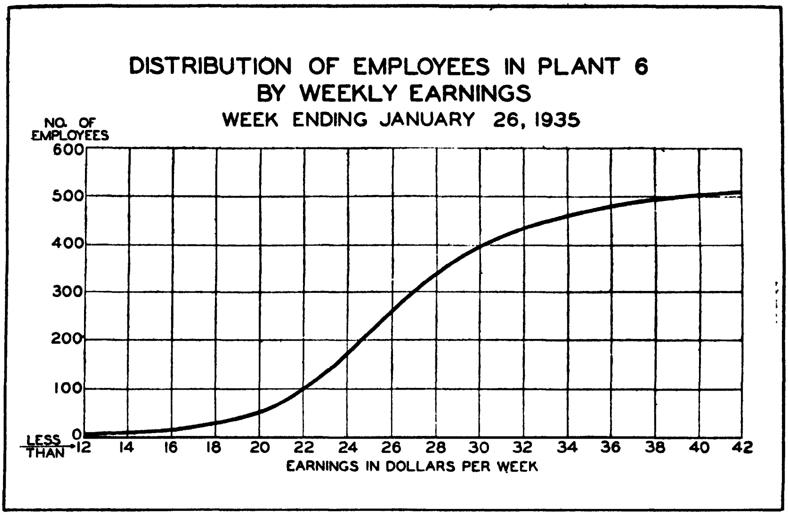


EXHIBIT 160.—A "less than" cumulative frequency curve. (See table in Exhibit 158.)

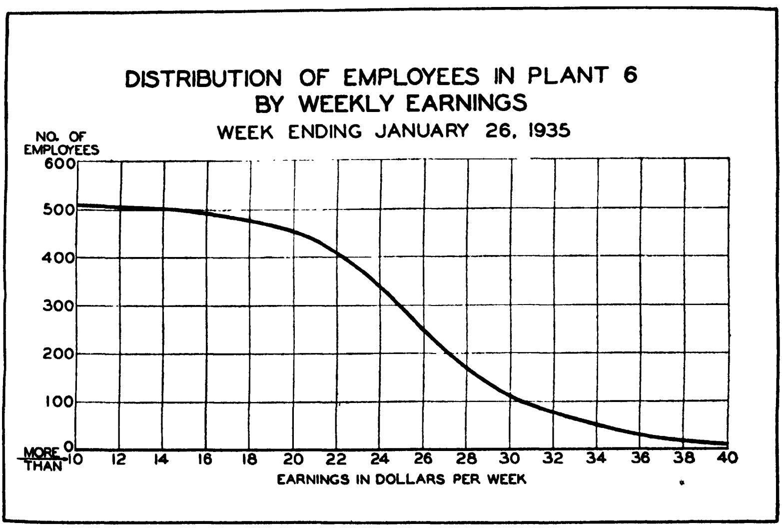


EXHIBIT 161.—A "more than" cumulative frequency curve. (See table in Exhibit 158.)

together so that each class includes all the lower or upper classes, depending upon from which end of the range the cumulating process is begun (see the last two columns of the table in Exhibit 158).

The cumulating process may begin at either end of the range. If it proceeds from the lesser to the greater, the corresponding frequencies are read "less than," as in Exhibit 160, and, if it proceeds from the greater to the lesser, the frequencies are read

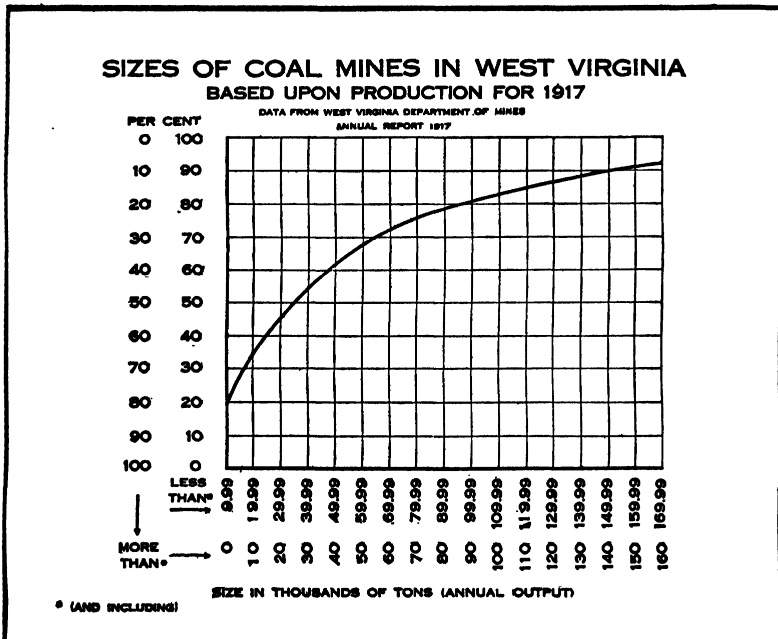


EXHIBIT 162.—A combination "less than" and "more than" cumulative frequency curve.

"more than," as in Exhibit 161. In a table like the one in Exhibit 158, when cumulations are read "less than" they refer to the upper limits of the classes, and when they are read "more than" they refer to the lower limits. It should be noted also that the limits are included when they are expressed as in Exhibits 161, 162, and 163.

Note also that a curve of the "less than" form extends from the lower left to the upper right, and that a "more than" curve extends from the upper left to the lower right, as illustrated in

Exhibits 160 and 161. In either case, as the curve approaches the vertical, relatively higher frequencies are represented.

A graph may be made to read both ways by including two sets of scales on the diagram, as illustrated in Exhibit 162, in which one of the vertical scales is read from the bottom up, and the other is read from the top down.

Both continuous and broken series may be plotted cumulatively, but the fundamental differences, which were discussed

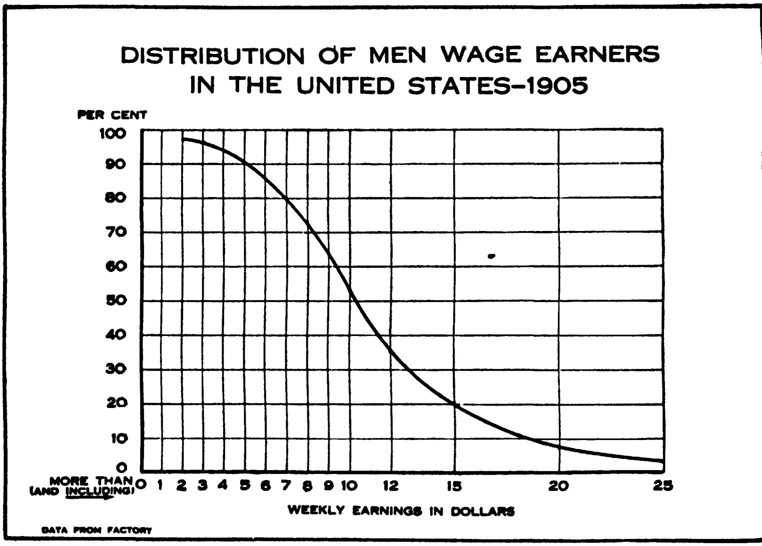


Exhibit 163.—A cumulative frequency curve with unequal class intervals.

in connection with the simple frequency curves, must be kept in mind. The discussion on the use of smoothed and broken curves for showing simple frequency distributions applies equally well to cumulative frequencies.

Exhibit 163 shows a cumulative curve plotted from data which were not given in classes of equal widths, because over part of the range the frequencies varied so rapidly. Though not all intervals are of the same size, the curve is so plotted that equal spaces represent equal values and, therefore, the curve is not distorted or misleading.

Comparisons of Frequency Distributions.—When two or more frequency distributions are to be compared, they are usually plotted on the same chart, and it is best to use either a broken or

a smoothed curve, according to the nature of the series, rather than a column diagram or bar chart.

A comparison of two distributions is illustrated in Exhibit 165, which shows the curves plotted according to the actual numbers

DISTRIBUTION OF MALE EMPLOYEES IN PLANTS 2 AND 3 BY AGES
Week ending June 8, 1935

Age to nearest year	Plant 2		Plant 3	
	Number	Per cent	Number	Per cent
15	17	0.1	0	0
16	120	1.0	0	0
17	426	3.4	0	0
18	763	6.2	0	0
19	1,020	8.3	0	0
20	1,380	11.2	7	.2
21	1,500	12.2	83	2.1
22	1,410	11.4	167	4.3
23	1,139	9.2	278	7.3
24	980	7.9	302	7.9
25	860	7.0	421	11.0
26	719	5.8	562	14.7
27	600	4.9	607	15.9
28	516	4.2	411	10.7
29	380	3.1	301	7.9
30	231	1.9	243	6.3
31	106	.9	199	5.2
32	72	6	149	3.9
33	61	.5	70	1.8
34	26	.2	26	.7
35	0	0	2	.1
Total	12,326	100.0	3,828	100.0

EXHIBIT 164.—Percentage distributions of frequency series. (See Exhibits 165 and 166.)

of occurrences. It will be noticed that the differences in the altitudes of the curves interfere with the comparisons of the distributions. Often it is advisable to place the large and the small groups on a more comparable basis by reducing the frequencies to percentages, as in the table of Exhibit 164, and to plot as shown in Exhibit 166, in which the total number of employees in each plant is taken as 100 per cent, and the percentage for each class

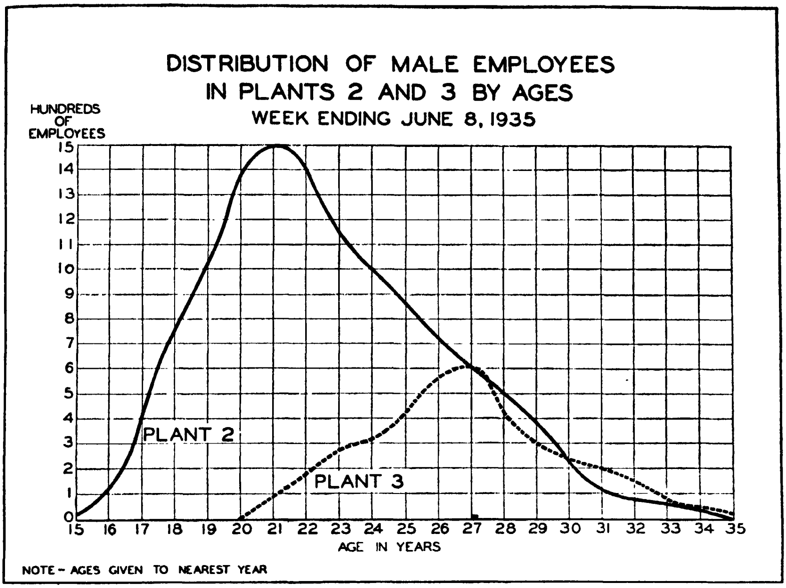


EXHIBIT 165.—Comparison of frequency distributions. (See Exhibit 166.)

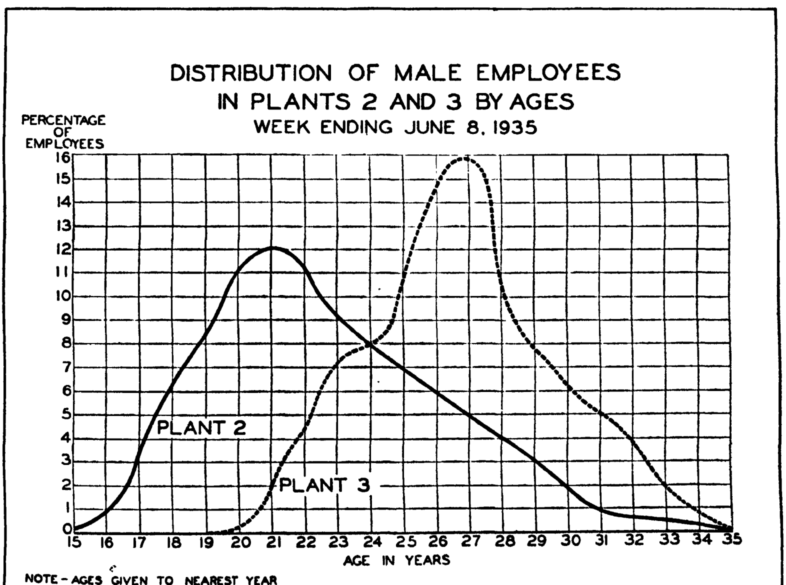


EXHIBIT 166.—Comparison of frequency curves on a percentage basis. (Compare with Exhibit 165.)

represents the proportion that the corresponding number is of the total.

In analyzing the curves in Exhibit 165, it would be easy to assume that the proportion of employees of twenty-one years of age in Plant 2 is greater than the proportion that is twenty-seven years of age in Plant 3. That the opposite is true, however,

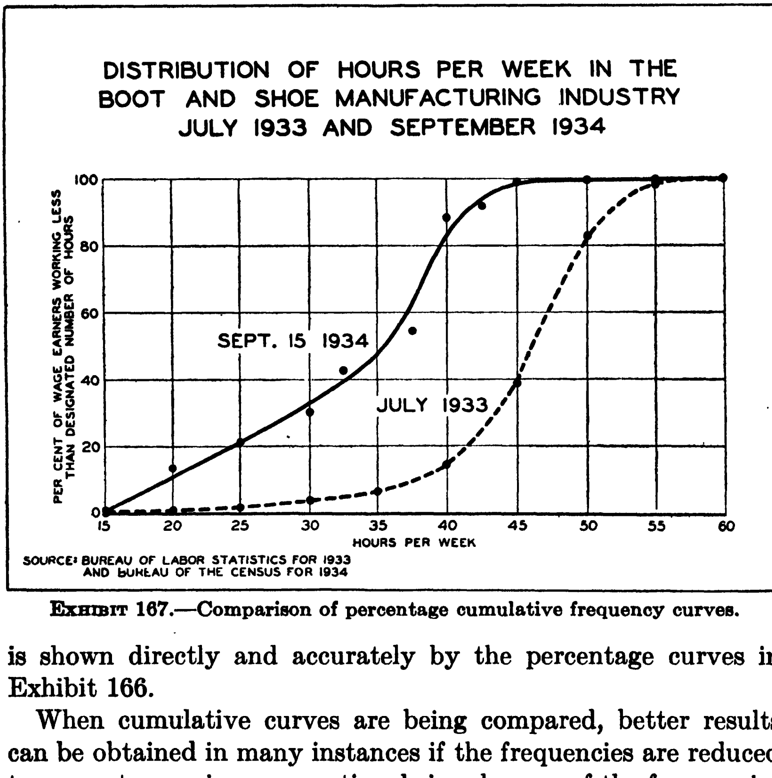


Exhibit 167.—Comparison of percentage cumulative frequency curves.

is shown directly and accurately by the percentage curves in Exhibit 166.

When cumulative curves are being compared, better results can be obtained in many instances if the frequencies are reduced to percentages, since proportional size changes of the frequencies and the regularity of the distribution can be appreciated more readily. The change in the proportions of employees working designated lengths of weeks in the boot and shoe manufacturing industry from July 1933 to September 1934 is clearly shown by the percentage cumulative frequency curves presented in Exhibit 167. The dots on the chart show the actual frequencies and the curves are smoothed to show tendencies.

CHAPTER VII

CORRELATION CHARTS

Graphic methods are a great aid to business men in their constant study of such relationships as the degree to which advertising can be depended upon to increase sales, the effect of population changes upon the markets for their products, the relation between the average balance maintained by a bank's depositors and its profits, the relation of the price of cotton to the size of the crop, the variation in the production of workers with the length of their experience, or the relation between wage rates and output, which are, of course, problems in correlation analysis. The analysis of business problems is a continuous process of correlation, and the successful business executive is an expert at practical correlation analysis. To describe the practical graphic methods of expressing such correlations as the foregoing is the object of this chapter.

Historical Curves and Scatter Diagrams in Correlation.—In view of the dominant position occupied by time series in business statistics, it is necessary at the outset to understand clearly the difference between the two commonly used graphic methods of presenting time series correlation. The data of Part *A* of Exhibit 168, showing the number of rainy days and the number of umbrella sales for each of 12 months, will be used to illustrate the two methods.

One graphic method of showing the correlation between the rainy day and umbrella series tabulated in Part *A* of Exhibit 168 is to plot the data as two historical curves as in Part *B*. The movements of the two curves correspond very closely (which indicates a high degree of correlation) until December, when there is a wide divergence because of Christmas sales. This method of presenting time series is the same as that described previously in this book (Chapters IV and V).

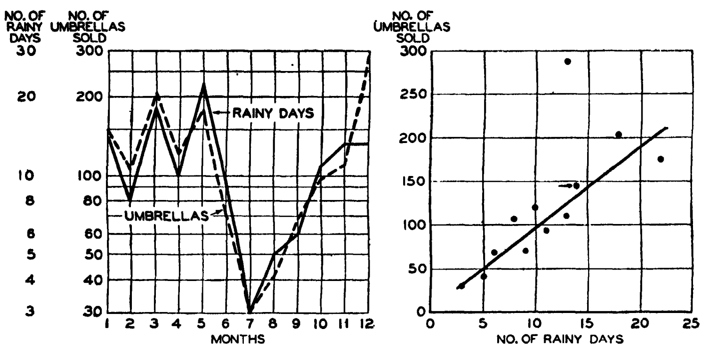
The other graphic method of showing correlation, known as the "scatter diagram," may be used to show the correlation between

rainy days and umbrella sales as in Part *C* of Exhibit 168. In this diagram the number of rainy days is plotted on the horizontal scale (x-axis), and the number of umbrellas sold is plotted on the vertical scale (y-axis). That is, to plot the data for January (14 rainy days and 147 umbrellas), one would find 14 on the rainy-day scale and plot this value opposite 147 on the umbrella scale (see Part *C*, Exhibit 168, in which the dot representing this

NUMBER OF RAINY DAYS AND UMBRELLAS SOLD

MONTH	NO. OF RAINY DAYS	NO. OF UMBRELLAS SOLD	MONTH	NO. OF RAINY DAYS	NO. OF UMBRELLAS SOLD
JANUARY	14	147	JULY	3	30
FEBRUARY	8	108	AUGUST	5	41
MARCH	18	203	SEPTEMBER	6	68
APRIL	10	120	OCTOBER	11	96
MAY	22	175	NOVEMBER	13	110
JUNE	9	70	DECEMBER	13	289

PART A.-TABLE OF DATA



PART B.-HISTORICAL CURVES

PART C.-SCATTER DIAGRAM

EXHIBIT 168.—Two graphic methods of comparing series in correlation analysis.

particular point is designated by an arrow). When the corresponding values of two series are paired in this manner, the relation between the two is indicated. The reader of Part *C* of Exhibit 168 will recognize rainy days as the independent causal factor, and the effect is shown in the dependent fluctuations of umbrella sales. This chart clearly indicates that, as the number of rainy days per month increases, the sales of umbrellas tend to increase. The trend line through the points helps to indicate this tendency. One marked exception to this tendency appears on the chart, however, in that one dot (near the top of the graph) is far out of line because of Christmas sales, as mentioned previously.

Primary and Secondary Correlation.—The scatter diagram and historical curve methods of presenting correlation are sometimes designated as *primary* and *secondary* correlation methods, respectively.

The method illustrated in Part *C* of Exhibit 168 and in Exhibit 169 is *primary* correlation. The chart in Exhibit 169 is similar to that in Part *C* of Exhibit 168, except that the relationships are expressed directly by a curve and no dots are shown. In both charts the variables are paired directly, and the relation-

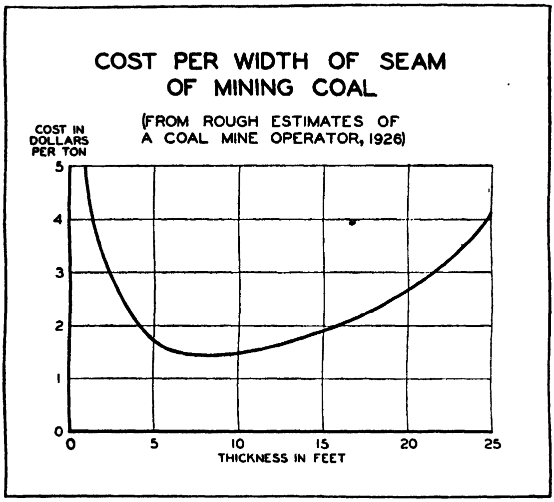


Exhibit 169.—Primary correlation.

ship is determined irrespective of any relationship which may exist between each and a common third factor. For instance, in Exhibit 169 the thicknesses of the different seams of coal are measured on the horizontal scale, while the costs of mining these different thicknesses are measured on the vertical scale, and the relation of the costs of mining to thicknesses of seams is shown by the resulting curve. Since the thickness of the seam is the standard by which we consider costs, it is the "independent variable" and is measured by the horizontal scale. Depending upon the thickness of the seams, the cost of mining will vary and this, the "dependent variable," is measured by the vertical scale.[1]

[1] By convention, the "independent variable" is plotted on the horizontal scale and the "dependent variable" is plotted on the vertical scale in all

Usually a correlation chart of this type is made as a curve, but occasionally it is made as a vertical bar chart when the series is not continuous.

The *secondary* correlation method of determining the association or relationship between two variables is illustrated in Part *B*

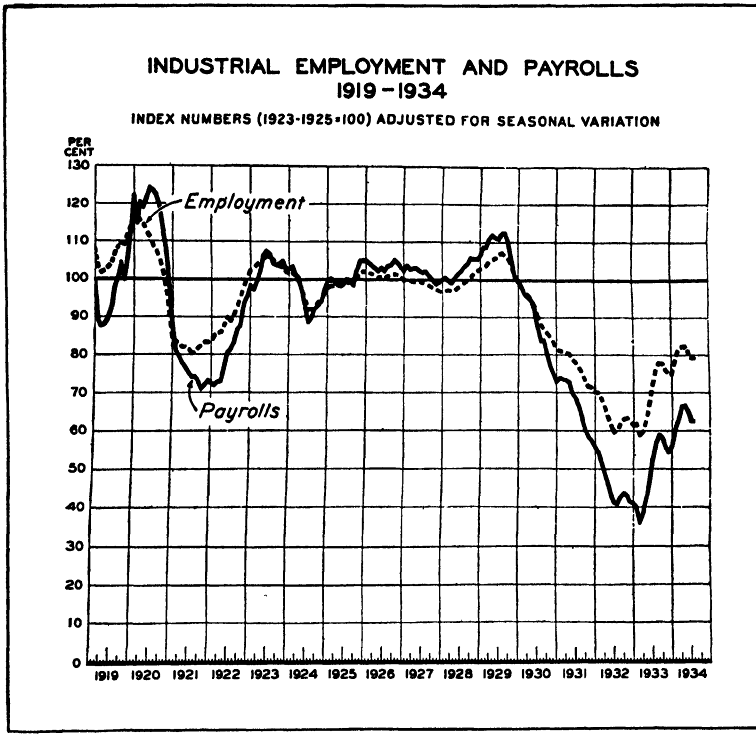


EXHIBIT 170.—Secondary correlation between two time series. (*Adapted from United States Bureau of Labor Statistics.*)

of Exhibit 168 and in Exhibit 170. By this method the relationship is determined by first relating each variable to a common third variable (time, in the two cases illustrated). Thus, in primary correlation the direct causal relationship between the

statistical charts. The horizontal scale defines the questions and the vertical shows the answer. One determines the independent variable when he decides upon the terms of his problem. The readers of charts like Exhibit 169, for instance, will recognize thickness of the seams as the basis of the study but will fix their immediate attention upon the dependent variable, or costs of mining, measured upon the vertical scale.

variables is indicated, while in secondary correlation the relationship between the two variables is determined through the relationship of each variable to a common third causal factor. Care must be taken, however, when studying correlation by the secondary method. While there may be a close correspondence between the two curves, there may be no causal relation whatever, and hence no correlation. Although curves drawn in this way may appear to correspond closely except for occasional

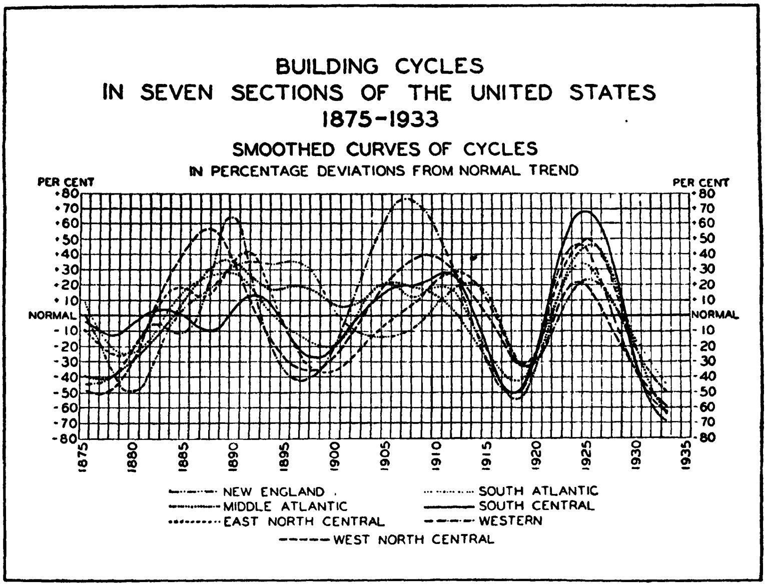


Exhibit 171.—Secondary correlation of a group of time series.

differences, yet these few differences may mean a great variation in the degree of correlation.[1]

[1] The degree of correlation may be determined mathematically by means of the coefficient of correlation. The student should be warned, however, against spurious correlation. No mathematical calculation is complete without an estimate of the probable error in the coefficient of correlation. On the whole, carefully handled and interpreted graphics have some advantages over mathematical correlation. The elements of judgment and special knowledge of conditions can be introduced in reading graphical correlation. Mathematical correlation, on the other hand, is often much too rigid for the data at hand. For further information on the coefficient of correlation, see any standard text on statistics such as those listed in the footnote on page 133.

When a graphic correlation study is to be made of a group of time series, the secondary method is generally used in practical business problems. Exhibit 171 shows the relation of seven different curves to each other. Another presentation of part of this same problem (in broken curves) is shown in Exhibit 102 on page 87. The primary method might be used in charting a problem like that of Exhibit 171 by making, say, the average the independent variable (horizontal scale) and then plotting

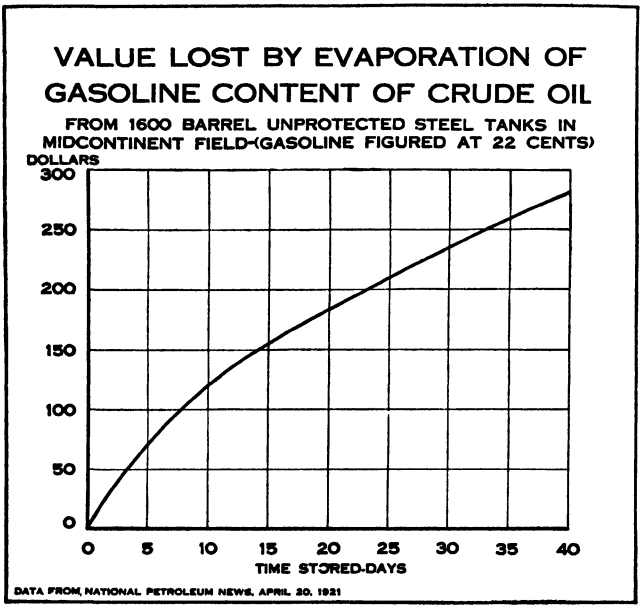


EXHIBIT 172.—A time series but not historical.

each series against this average, but ordinarily the secondary method (Exhibits 102 and 171) is the most easily understood by business men when more than two series are involved.

Historical Time Series and Frequency Series Distinguished from Other Correlative Series.—In a general sense any graph with horizontal and vertical scales, in which the corresponding values of two variables are represented by a single point, is a correlation curve. However, since in business statistics historical graphs covering periods of definite dates are used probably more than all other kinds combined, they have been treated separately in Chapters IV and V. It should be remembered that

a *historical* graph is a graph of *dates*. Not only is a period of time expressed along the horizontal axis, but this period represents definite dates, such as certain months or years. In the correlation chart shown in Exhibit 172, a period of time is represented by the horizontal scale, and in Exhibit 180 (right-hand part), page 170, time is represented by the vertical scale, but these are not historical graphs, since the time periods are not represented by definite dates. Only the lengths of the periods are important in these charts, and dates would not be significant.

Another important kind of graph, the frequency graph, has been considered separately in the preceding chapter (Chapter VI). The frequency graph shows for each class the *number* of occurrences. The correlation curve shows the effect of the independent variable upon the dependent variable or, for instance, the effect of the width of seams upon the cost of mining coal (Exhibit 169).

Correlation Chart Scales.—In laying out a correlation chart, the horizontal and vertical scales should be so chosen that the space required for the range of the data will be approximately equal for both series. If this is not done, there may be difficulty in determining even the approximate degree of correlation. In Exhibit 173, Part *A* illustrates how the scales should be chosen, while Part *B* illustrates how they should not be chosen.

The "line of best fit" through the dots in Part *A*, Exhibit 173, approximates an angle of 45 degrees and the dots are in a narrow band along this line.[1] In Part *B* the dots are in a still narrower band, which, of course, does not indicate higher correlation but is simply the result of the lesser slope, caused by the horizontal scale being too great in proportion to the vertical scale.

One of the simplest methods of laying out the scales, so that the line of best fit (if there is one) will approximate a 45-degree angle, is to have the ratio between the scales inversely proportional to the ratio of the ranges of the data. That is, if the range of the x data is 220 million bushels and the range of the y data is 20 million acres, then the scale for x would be to the scale

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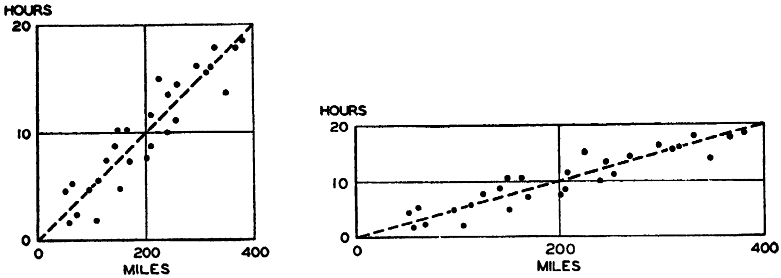
[1] "Lines of best fit" may be mathematically computed by the method of least squares. In Exhibit 173, however, the lines were drawn in freehand. This method of "fitting the line by inspection" is sufficiently accurate for most practical business problems, and it facilitates a judicious weighting of the observations.

for y in the ratio of 220:20, or 11:1. A convenient approximation is usually satisfactory, which in the preceding case might be 10:1, so that the same linear distance would represent 10 bushels for x as would represent 1 acre for y.



A.—Correct. *B*.—Incorrect.

Exhibit 173.—Relations of vertical to horizontal scales on correlation charts.

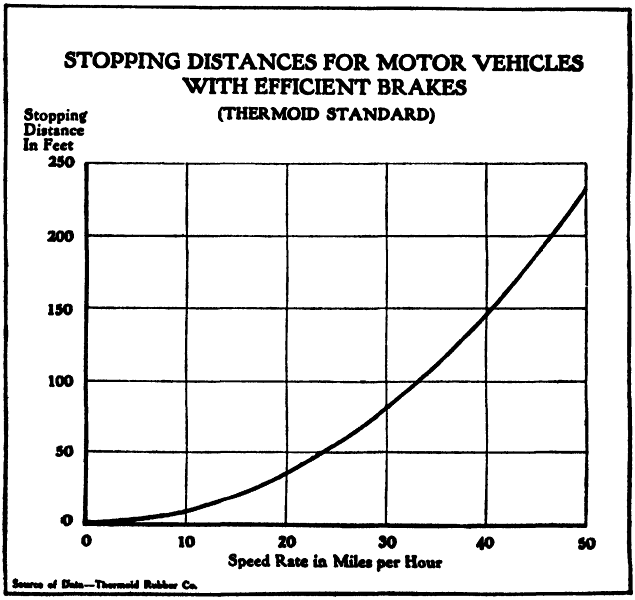


Exhibit 174.—When showing a continuous series, the curve should be smoothed.

Smoothed and Broken Curves.—If the nature of the data is such that changes in the curve are gradual, with possible values at all points on the curve, the smoothed curve should be used (Exhibit 174). Thus, on this curve there is a stopping distance for every possible speed rate up to 50 miles per hour; and as the

speed rate gradually increases, the stopping distance gradually becomes longer.

If, on the other hand, there cannot be values for every point, the broken curve should be used (Exhibit 175). In such instances the plotted points represent values, but all that the connecting lines do is to point the direction from one value to another, thus helping the reader to gain a general impression of the distribution. There cannot be values between points in Exhibit 175, since motor trucks, according to the sample, are made only in the sizes designated. If a bar chart is ever used in correlation charting,

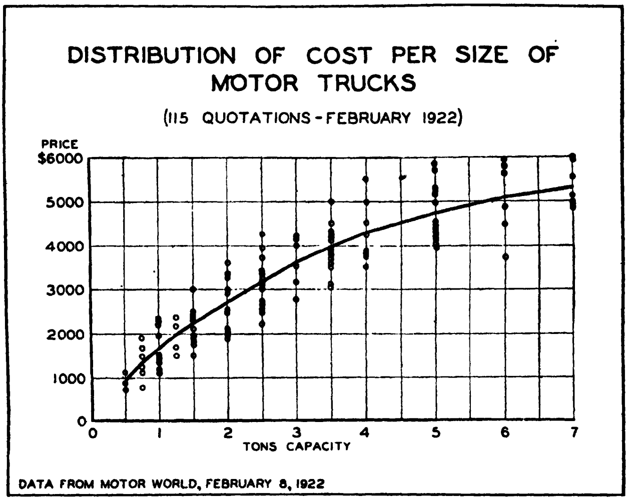


EXHIBIT 175.—When plotting a broken series, the points should be connected with straight lines.

it should be used for the kind of distribution shown in Exhibit 175 rather than for the kind shown in Exhibit 174.

Position of the Curve.—In fitting a curve to a large number of scattered points (Exhibits 175 and 176), care must be taken to have each portion of the curve as nearly as possible at the center of gravity of the dots in any vertical section of the chart. A curve may not be correctly located by having the same number of dots on each side, for their distances from the curve must be considered. As a simple rule to be kept in mind when fitting a curve, one may consider the dots in any vertical section of the graph as weights arranged on a horizontal lever, with the curve as

the fulcrum, and then shift the curve so that the lever will balance (Exhibits 175 and 176).

Indicating Degree and Type of Correlation Graphically.—The degree and type of correlation may be indicated for time series by both the historical curve and scatter diagram methods. With reference to Exhibit 177, the historical curve charts show a high degree of direct correlation in Part *A*, a high degree of inverse correlation in Part *B*, and very low or no correlation

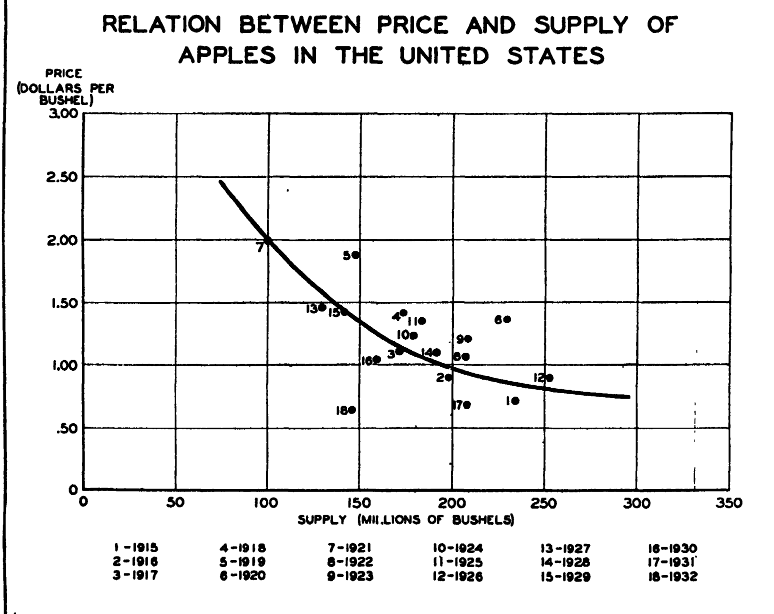
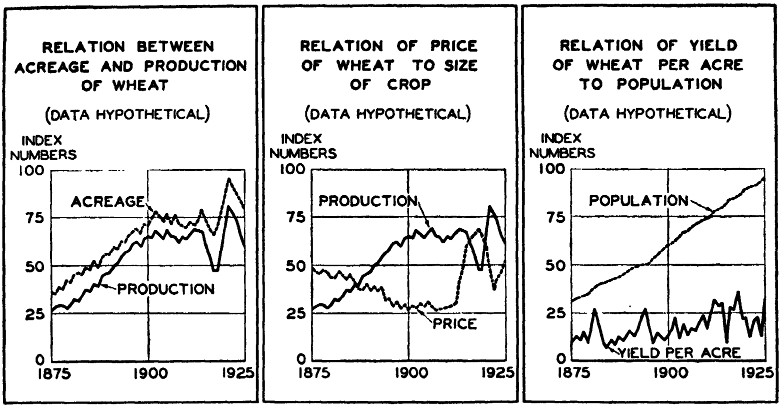


EXHIBIT 176.—Fitting a curve to scattered points.

in Part *C*. The scatter diagrams in Exhibit 178 show a high degree of direct correlation in Part *A* (narrow band of dots running from the lower left to the upper right), a high degree of inverse correlation in Part *B* (narrow band of dots running from the upper left to the lower right), and very low or no correlation in Part *C* (dots widely scattered).

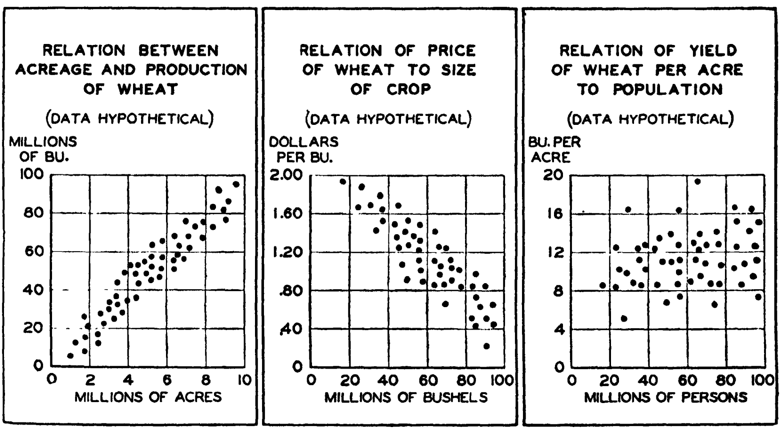
As indicated previously, in making correlation analyses of historical curves, great care should be taken that any instances of close correspondence, that do not represent actual correlation,

be recognized. Spurious correlation is often found between two time series, and frequently it can be made very high by intro-



A.—High direct correlation. *B.*—High inverse correlation. *C.*—Low or no correlation.

EXHIBIT 177.—Historical curves indicating type and degree of correlation.



A.—High direct correlation. *B.*—High inverse correlation. *C.*—Low or no correlation.

EXHIBIT 178.—Scatter diagrams indicating type and degree of correlation.

ducing the element of time lag, particularly when the series are short.[1]

[1] See M. C. Rorty, "Statistics and the Scientific Method," Presidential Address, Ninety-second Annual Meeting, American Statistical Association, December 31, 1930, published in the *Journal of the American Statistical Association*, pp. 1–10, March, 1931.

The scatter diagram, when used as a means of estimating the degree of correlation, possesses the very desirable advantage that it is no more influenced by items of extreme size than by items of usual or average size. Some of the mathematical methods, however, are decidedly influenced by extreme items. In fact, an item of extreme variation may have such influence as to give a mathematical expression of low correlation when there really is high correlation, or of high correlation when there really is low correlation.

Estimating from a Correlation Curve.—If a primary correlation curve or line of best fit truly represents a high degree of correlation between two series, it is possible to estimate any value in one of the series from the corresponding known value in the other. Suppose, for example, that we wish to know what $1 would amount to in 30 years at 6 per cent compound interest. Referring to the upper curve in Exhibit 181 (either Part *A* or Part *B*), we note that at 30 years (horizontal scale) the curve has a value of approximately $6 on the vertical scale. In this assumption, the known figure is 30 years, and upon it we base our question. Conversely, we might wish to know how many years it would take for $1 to increase to $6. We could then find 6 on the vertical scale (Exhibit 181) and under this value on the curve read the corresponding number of years (approximately 30). When charts are made especially for such purposes, however, they are designed so that what is ordinarily the independent variable is laid out as the horizontal scale (see footnote on page 159). This method of estimating (as described in connection with Exhibit 181) may be applied similarly to the other primary correlation charts presented in this chapter.

Comparison of Correlation Curves.—Exhibits 179 and 180 are examples of the comparison of correlations. Exhibit 179 shows the differences in the costs of the gas and electric arc methods for cutting steel for any thickness within the extremes of the chart. Exhibit 180 compares the differences in costs and time used, in relation to distance, in transportation by railroads and motor trucks.

When comparisons such as Exhibits 179 and 180 are made, the units of the scales must be common to all of the series that are plotted. For instance, in Exhibit 179 if thicknesses for the gas method are measured in inches, then thicknesses for the elec-

tric arc method must be measured in inches; and if the cost of gas is measured in cents per foot of steel, then the cost of electricity must be measured in the same way.

It will be noted that the points are widely scattered in the "Railroad" curve of the right-hand part of Exhibit 180, and hence the degree of correlation is very low. When curves are

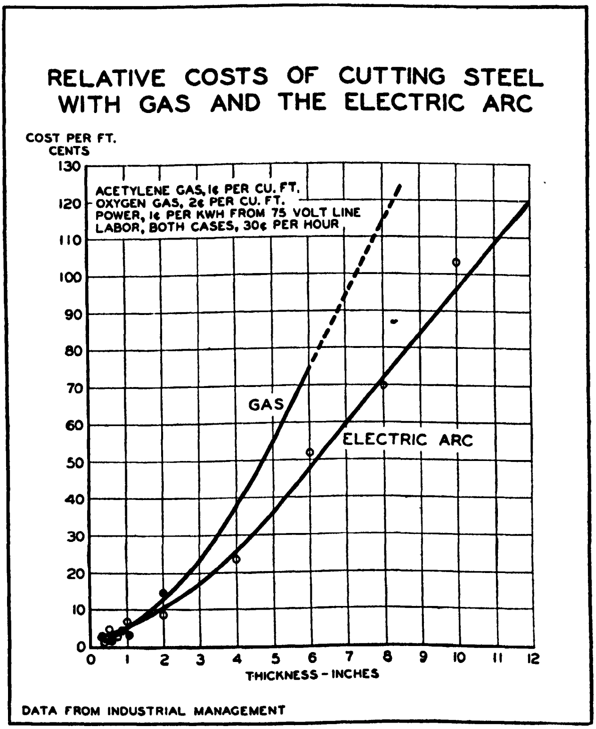


Exhibit 179.—Comparison of correlation curves. (*Adapted from Industrial Management.*)

freely smoothed, the actual plotted points should be indicated clearly as shown in this chart.

The charts in Exhibits 179 and 180 were drawn on arithmetic vertical scales, because they were intended to show the *differences in the sizes* of the costs and time periods involved. If, however, not the *size differences* but the *rates of change* are to be compared, logarithmic scales should be used.

Use of the Logarithmic Scale.—Exhibit 181 shows how an investment of $1 would increase at 4, 5, and 6 per cent compound

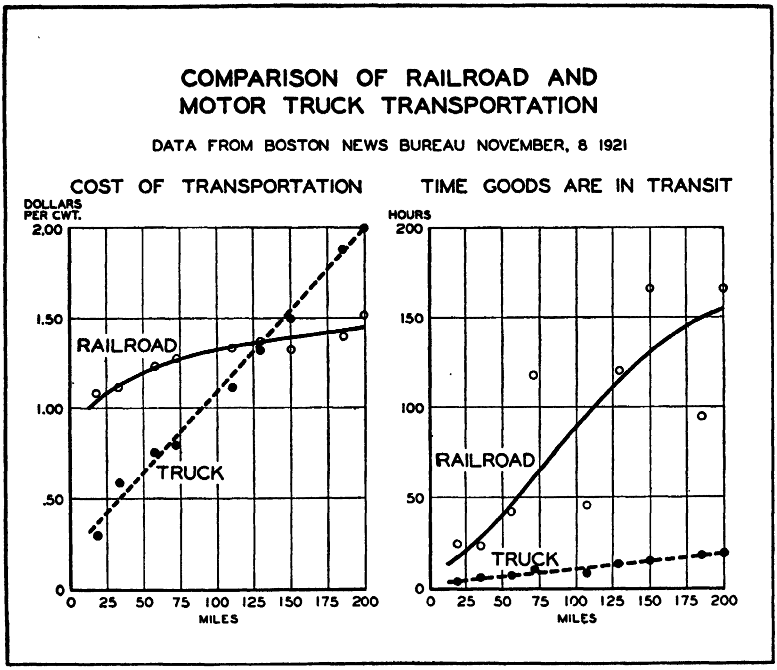


EXHIBIT 180.—Comparison of correlation curves.

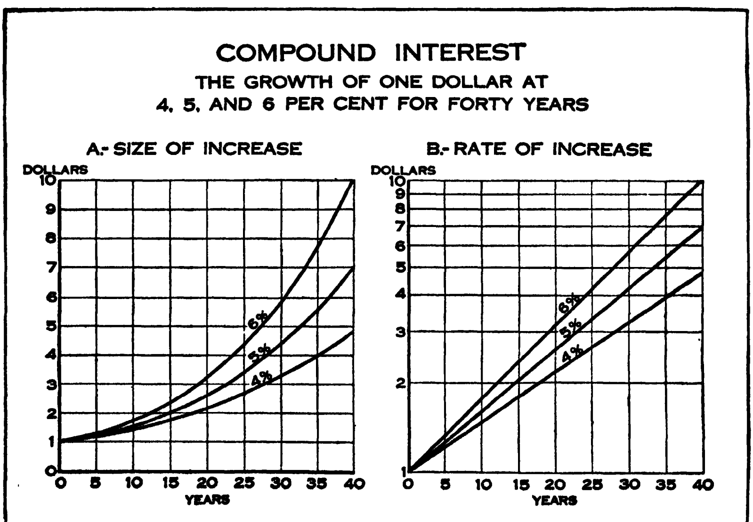


EXHIBIT 181.—Arithmetic and logarithmic scales. (Time series but not historical.)

interest up to 40 years. Part *A* of Exhibit 181 shows the curve plotted on an arithmetic scale, and its purpose is to show the size of the investment for any length of period up to 40 years. It does not show the rates of increase, and it does not indicate that the rates of increase are more rapid toward the end of the period. The rates of increase are constant, and this is shown by the straight lines on Part *B* of Exhibit 181, which is plotted to a logarithmic scale.

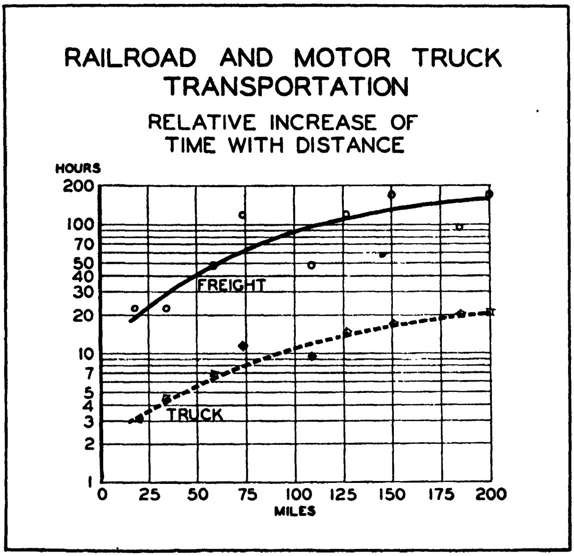


EXHIBIT 182.—Logarithmic correlation curves.

In reading the time chart in Exhibit 180 (right-hand chart), it must be understood that the purpose is to show differences and not ratios. The rates of change in the two curves as the mileage increases cannot be directly compared. In attempting to do this, one is likely to decide that, as the distance increases, the time taken by railroads increases at a much faster rate than that taken by trucks. But this is not the case, as is readily seen by plotting the curves on a logarithmic vertical scale as shown in Exhibit 182. As the logarithmic chart is described in detail in Chapter V, it will not be considered further at this point.

CHAPTER VIII

GEOGRAPHICAL DISTRIBUTIONS—STATISTICAL MAPS

When it is desired to show the geographical distributions of magnitudes, frequencies, or ratios, various kinds of statistical

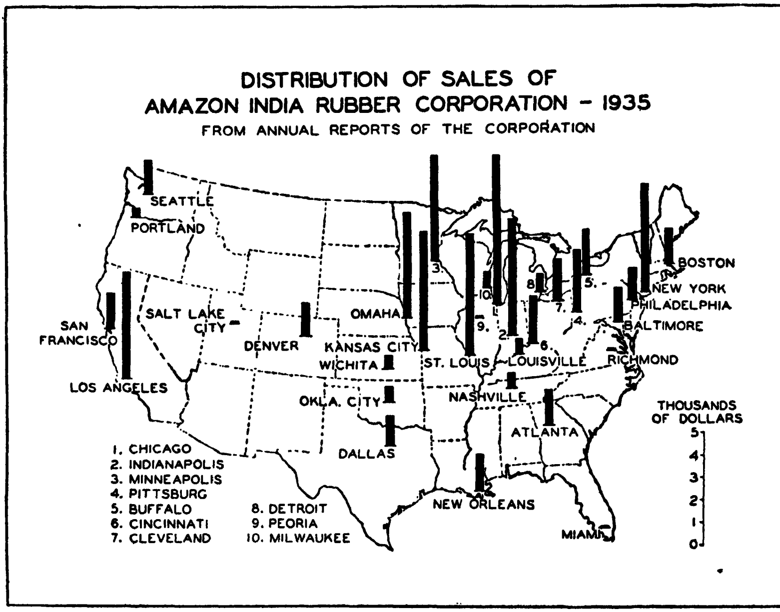


Exhibit 183.—Use of bars to show sizes at geographic points.

maps are used. The statistical map shows graphically the relative geographic positions of the facts represented.

Outline Maps.—Plain outline maps with but little or no coloring are the most satisfactory for statistical work.[1] Mountains, streams, roads, city and state names, and the like should not be shown unless they add a distinct value to the presentation.

[1] Satisfactory outline maps usually can be obtained from local stationery stores; Rand McNally and Co., Chicago; National Map Co., Indianapolis; Educational Exhibition Co., Providence; United States Department of the Interior, Washington; State Highway Commissions; City Engineers; and other government offices.

Methods of Representing Data.—Bars, circles or dots, and cross-hatched shades are widely used in making statistical maps. The discussions, in previous chapters, of the advantages and limitations of bars and circles apply equally well to the making of statistical maps, although the graphic problem differs to a considerable degree, as will be pointed out in the descriptions of the different types of maps in the present chapter.

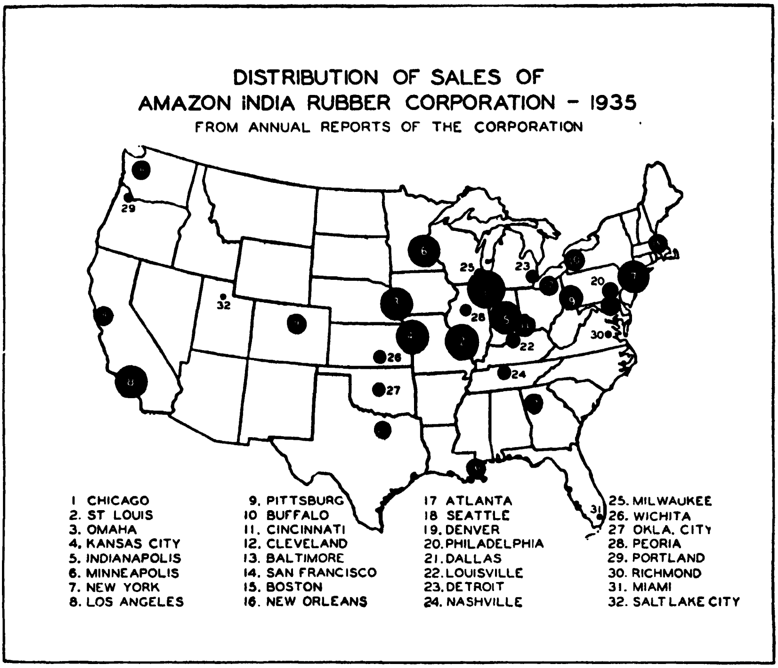


EXHIBIT 184.—Use of circular areas to show sizes at geographic points.

Simple Comparisons of Size for Geographic Points or Areas.—The most simple kind of statistical map is that which represents sizes in their geographic positions. At the outset it is well to appreciate the difference between maps which show sizes at points, and maps which show sizes for certain areas. For example, Exhibits 183, 184, and 186 represent magnitudes for points rather than for areas, and the maps show the distribution of these points, while Exhibits 185, 187, 188, and 189 represent sizes, not for points, but for certain defined areas. However, the

graphic methods used in these two types of maps are the same, and therefore they will be considered together.

In showing the geographical distribution of simple sizes, bars are sometimes used, as illustrated in Exhibit 183, in which the length of the bar is proportional to the size represented. This method shows directly much more regarding the distribution than would the ordinary bar chart, but sizes cannot be compared so easily since the bases of the bars are not in a straight line.

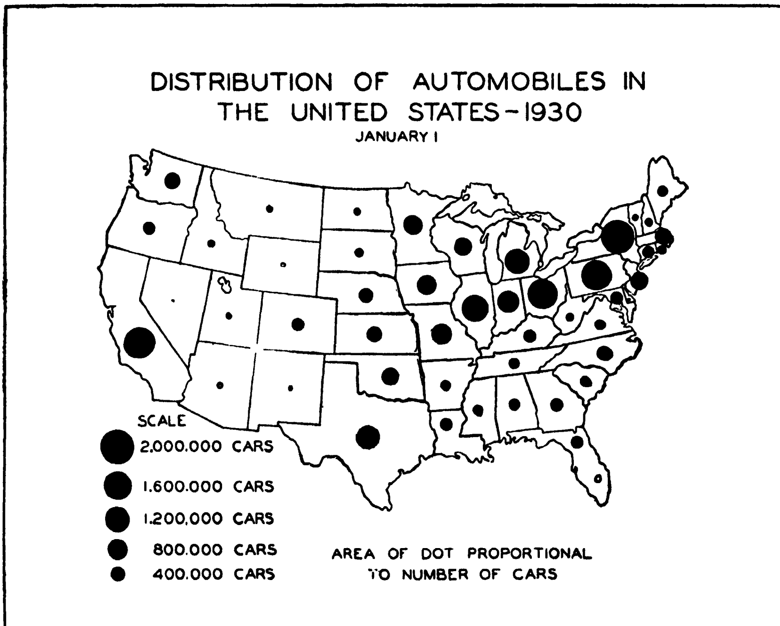


EXHIBIT 185.—Use of circular areas to show sizes by states.

When this method is used, all of the bars should be drawn to the same scale, and it is well to include a reference scale, as shown in Exhibit 183, to aid in making comparisons.

The geometric form most commonly used in showing data like that of Exhibit 183 is the circle. Either the circle or circular area is used, according to the distribution of the data. Circular areas or dots show up better than open circles when they can be used (Exhibits 184 and 185); but if there is much overlapping, open circles are the most clearly read (Exhibit 186). On all such maps as Exhibits 184, 185, and 186, the circles should

be so drawn that their areas are proportional to the sizes represented (see Exhibit 13 on page 20). A map on which the sizes of the dots vary gives a good general impression very quickly and easily, but it is not satisfactory for very close comparisons because circular areas are difficult to compare. To most persons, a circular area twice the size of another appears to be less than twice as large. In presentations like Exhibit 185 it is well to include a scale for reference as illustrated. For more exact

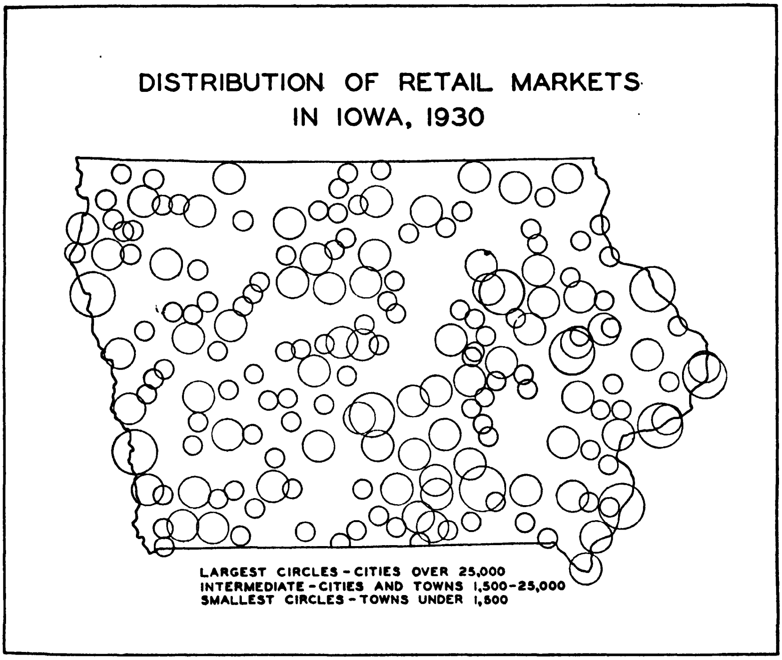


EXHIBIT 186.—Use of circles to show sizes at geographic points. (*Adapted from Woman's World.*)

reference in such charts, the actual numbers may be shown in small figures in the corners of the states. On a map like Exhibit 186 the limits of the class intervals should be stated definitely, and the circles should be drawn in proportion to the middle of each class. It should be noted that in this respect this map is faulty. (Other reasons entered into the determination of the circle sizes in the original from which Exhibit 186 is adapted.)

Another type of dot map is illustrated by Exhibit 187. Instead of using different-sized dots to indicate different values,

uniform sizes are used, and the number of dots for the different states is proportional to the number of cars represented. Thus, in Exhibit 187, one dot represents 200,000 automobiles. On this map, numbers are represented to the nearest 50,000 by using a quarter of a dot. The unit which one dot is to represent should be so chosen that in no case will there be so many dots that they cannot be counted at a glance, and in no case will significant data be omitted. The arrangement of the dots

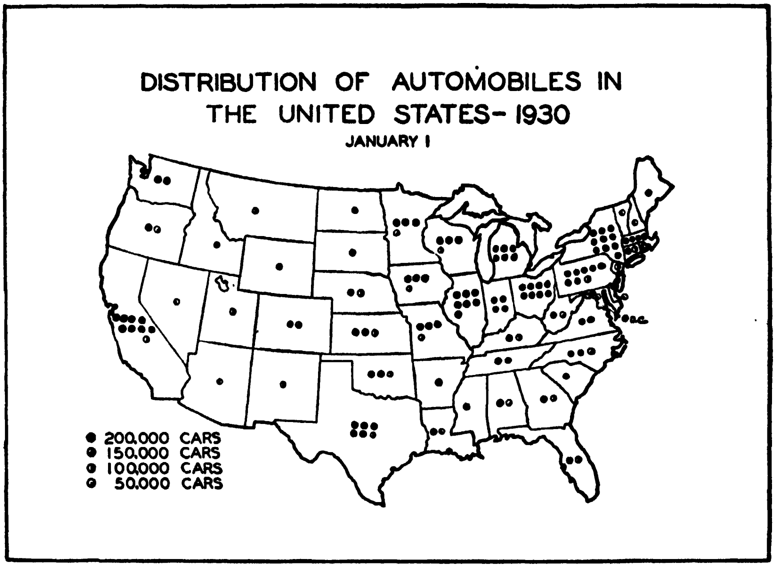


Exhibit 187.—Grouping dots to show sizes. (This type of map should not be confused with Exhibit 197, in which the primary purpose is to show density rather than size.)

should be such that the groups in the different states (or other unit areas) may be compared easily. They should be arranged squarely and in straight rows to facilitate counting, and they should be equally spaced in all areas so that the spaces covered by the dots will not cause inaccurate impressions. Thus ten dots spread over a larger space than that covered by twelve dots might impress the reader as representing the larger number. When this type of map is made, the purpose is to represent not *densities* but *sizes*, and it should not be confused with the density dot maps discussed on pages 184 to 187, inclusive.

The method illustrated in Exhibit 188 is similar to that of Exhibit 187, except that, instead of a row or group of dots, the length of the bar represents the numbers. Long bars may-be broken up into several parts so that they will fit into the proper area. The scale to which the bars are drawn should be small enough to allow the areas on the map to include the bars, and large enough to show all significant figures. This scale should always be shown for reference, as illustrated in Exhibit 188.

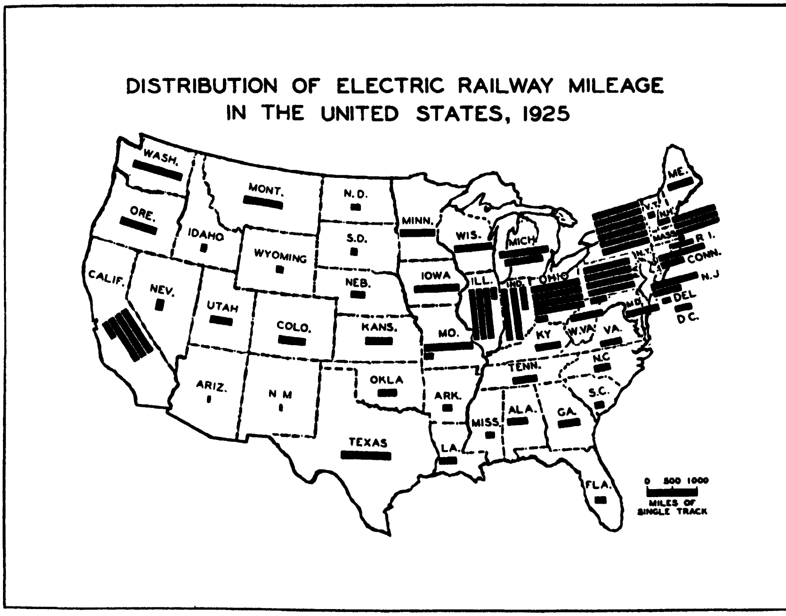


EXHIBIT 188.—Use of bars in showing geographical distribution of sizes. (*Adapted from ABP News.*)

Exhibit 189 illustrates an excellent method of showing geographical distributions of size, when it can be used. The columns, rising perpendicularly from the map, represent the different values. The map is mounted on a wood base. The long columns (dowels) are inserted in holes bored in the wood. The very short columns are either nailed or glued to the map.

Exhibit 190 is an effective method of showing the tariff duties of the different countries in Europe, the amounts of the duties being represented by the heights of the walls. Three-dimen-

sional maps of the general types illustrated by Exhibits 189 and 190 are very effective for use in commercial exhibits or in such

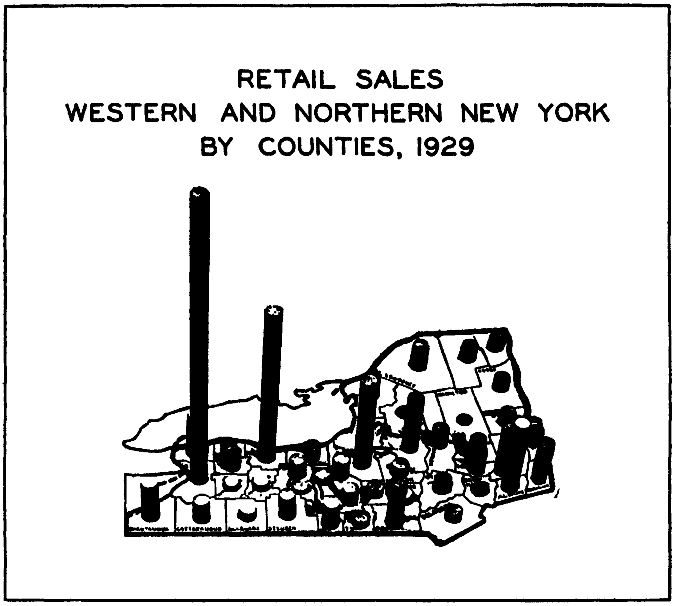


EXHIBIT 189.—Bar type of map made by mounting dowels, cut to the proper lengths, on a wood base. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

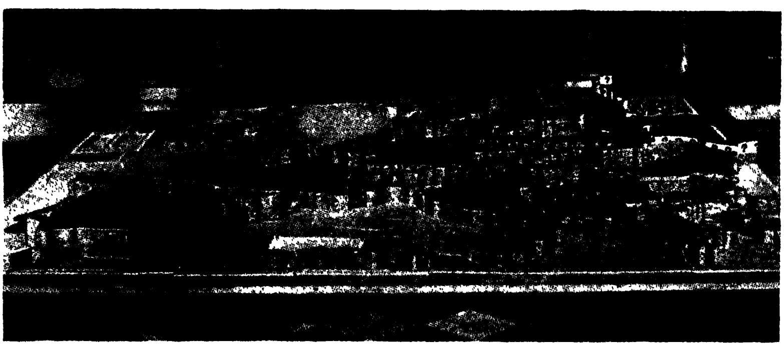


EXHIBIT 190.—"The Tariff Walls of Europe"—an effective method of indicating the tariff duties of the various European countries. (*Courtesy of Sir Clive Morrison-Bell.*)

displays as the one illustrated in Exhibit 3 on page 9. When the original (in three dimensions) cannot be used, as in reports

and publications, photographs can be made similar to Exhibits 189 and 190.

An effective method for certain types of popular presentations is illustrated in Exhibits 191 and 192. On these maps the areas for which the data are shown are raised to the proper heights from the base plane.

Exhibit 191 represents a photograph of a model, made with the aid of a jig-saw, while Exhibit 192 represents a handmade

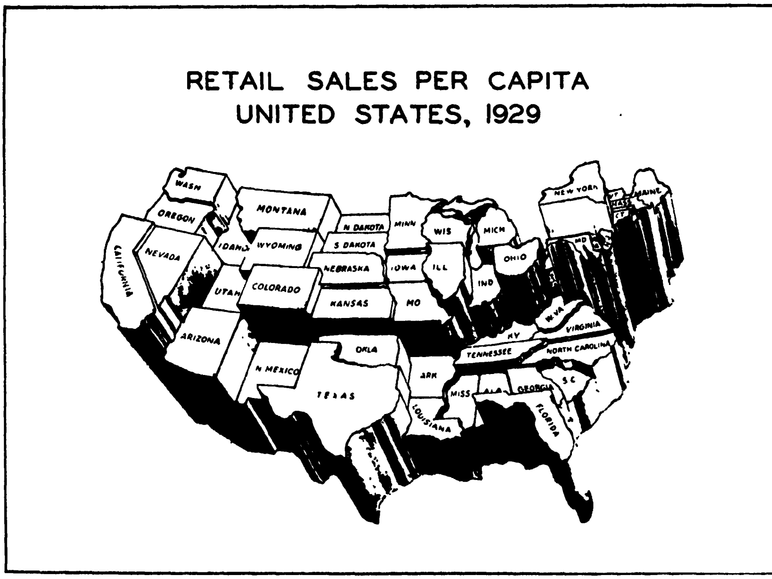


EXHIBIT 191.—Representing statistical data by raising states above the base plane of the map. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

drawing. In using such maps, care must be taken that distortions of the data do not result because of differences in the sizes of the areas (see page 183).

Exhibit 193 represents a method of showing data for states by drawing the area of the state on the map proportional to the numbers represented. Such maps, however, are not superior to the dot or bar maps just described (Exhibits 183 to 189, inclusive) for showing distributions of size. In many cases the method illustrated in Exhibit 193 would result in the states being so distorted that little if any resemblance of their true shapes would remain, and even their relative

positions would be inaccurate. It is much more difficult to compare the irregular areas on such maps than it is to compare either circles or bars.

Different shades of cross-hatching may be used to represent magnitudes on statistical maps. Thus, in Exhibit 194, the coal supply, in number of days, is represented for each shaded area by

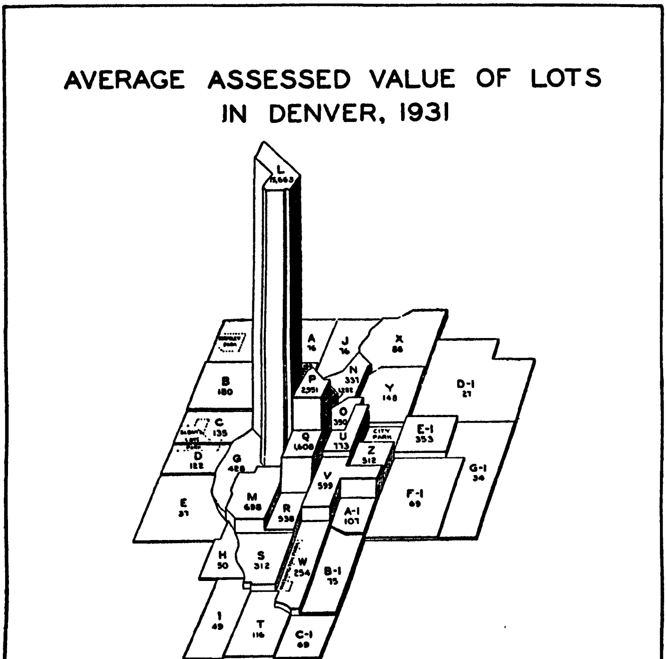


EXHIBIT 192.—Representing statistical data for a city by raising areas above the base plane of the map. (*Courtesy of University of Denver Bureau of Business Research.*)

different kinds of shading. In such cases of size comparisons, however, the bar and dot maps previously described present a clearer statistical picture of the situation. On a cross-hatched map of size comparisons, the shading ranges from white to black—the extreme shades representing the extremes of the data. The number of shades to be used depends upon the number of classes into which the data are divided, but ordinarily the lightest shading should represent the small extreme and the

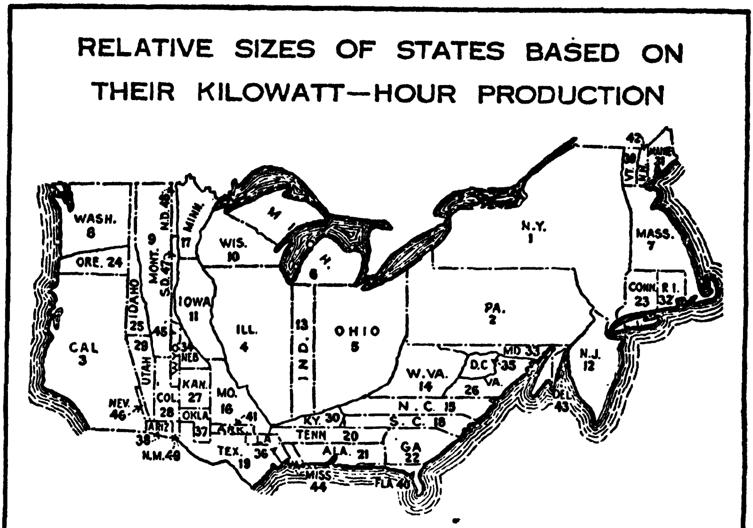


Exhibit 193.—States drawn in proportion to the production figures represented. This is not a good method for general use although occasionally it is quite effective. (*Courtesy of Mechanical Engineering.*)

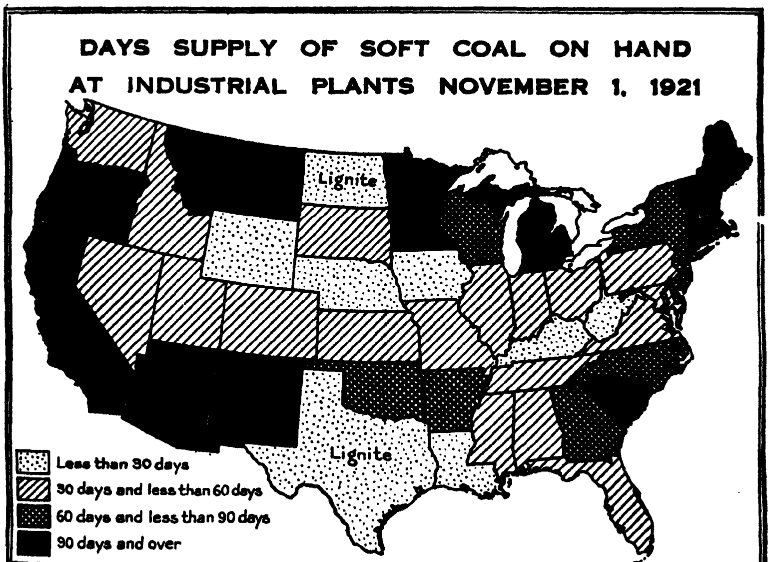


Exhibit 194.—Representing sizes by shading. (*Courtesy of United States Geological Survey.*)

darkest shading the large extreme. As a rule, these classes should be of uniform size, and the shades should vary to the same degree that the classes vary. In Exhibit 194 four different kinds of shading, varying from light to dark, are used. A clearer impression would be given if the shades varied by the same degrees of "blackness." That is, the lowest shading might be 0 per cent black; the second, 33⅓ per cent black; the third, 66⅔ per cent black; and the highest, 100 per cent black. (The shaded map

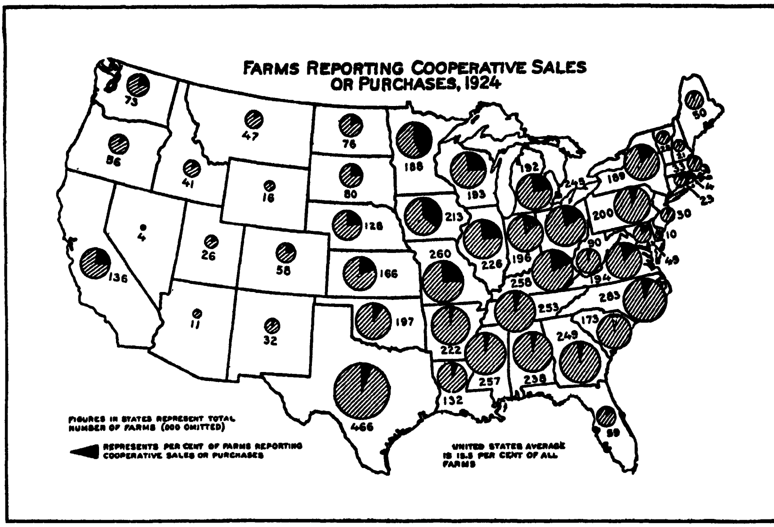


Exhibit 195.—Geographical distribution of aggregates showing their component parts. (*Courtesy of United States Department of Agriculture.*)

which shows *size comparisons* should not be confused with the shaded map which shows relationships where something other than area is a factor. These maps are described on pages 192 to 196, inclusive.)

A method of showing the geographical distribution of aggregates, with their component parts indicated, is illustrated in Exhibit 195. Each circular area is a pie chart drawn with its total area proportional to the number represented. Component bars could, of course, be used in a similar manner.

Exhibit 196 illustrates a good method for showing a series of observations for each point or unit of area. Similar little time series charts can be made in the curve form for such maps.

Exhibit 196 would be easier to read if the same spacing and the same range were used for the vertical scales of all of the graphs.

Care should be taken, in making maps like those illustrated in Exhibits 183 to 196, inclusive, that the variations in the areas for which the data are shown do not destroy the significances of the data themselves. Such maps compare data which have been collected on an area basis. These areas may be equal, like square miles; but they usually are unequal, like states,

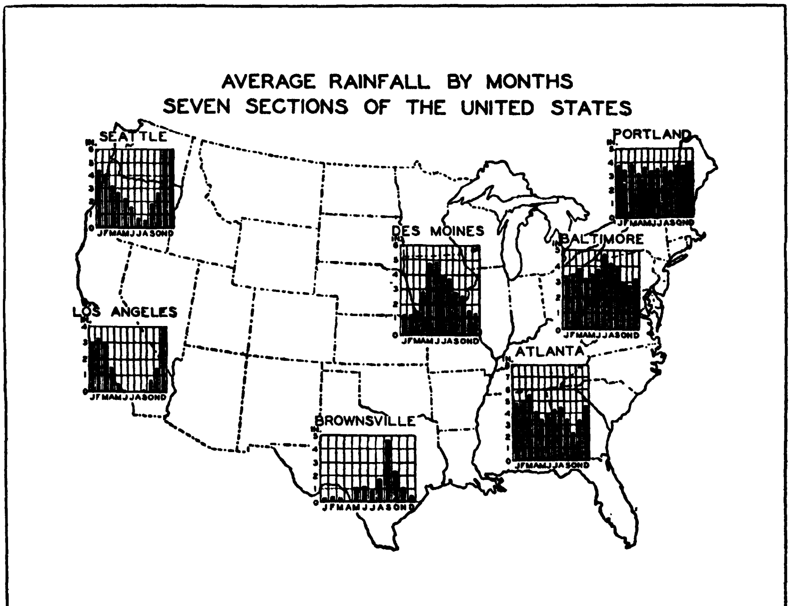


EXHIBIT 196.—Geographical distribution of time variables. (*Courtesy of Successful Farming.*)

counties, or cities. For instance, in making a map of the United States, it is often impractical to map the data on a basis of areas smaller than states, and, if the population figures of the different states are compared, unequal areas are compared also. That is, in comparing the sizes of the populations of Massachusetts and Missouri, we find that of Massachusetts almost one-half larger. However, this comparison may not be significant for the purpose at hand as the area of Missouri is about eight and one-half times that of Massachusetts, and the number of persons per square mile in Massachusetts is approximately

ten times that of Missouri. In such a case, some other method might have to be used. In problems of the type illustrated in Exhibits 183 to 196, inclusive, it is desired to show simply the numbers or sizes at geographic points or by unequal areas, such as states, with little regard for the densities of the distributions. That is, we may wish to compare numbers of automobiles by states (Exhibits 185 and 187) without showing how close together they are, and without regard for differences in area. If, on the

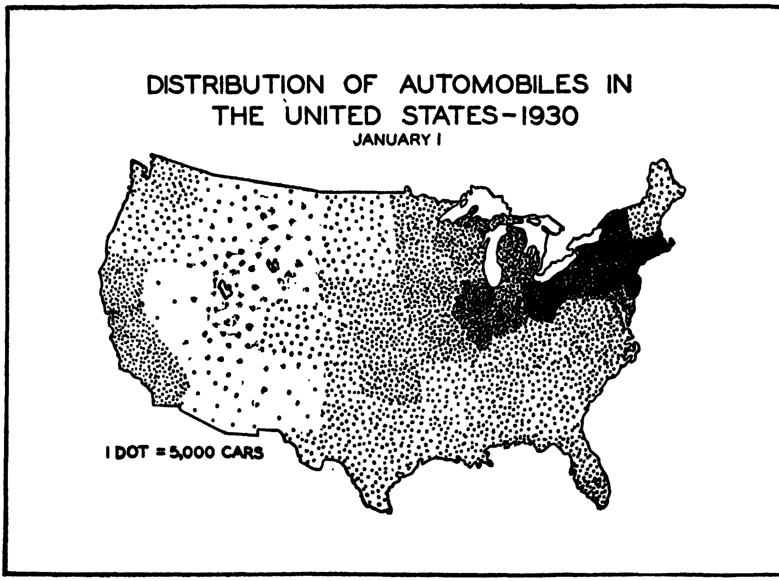


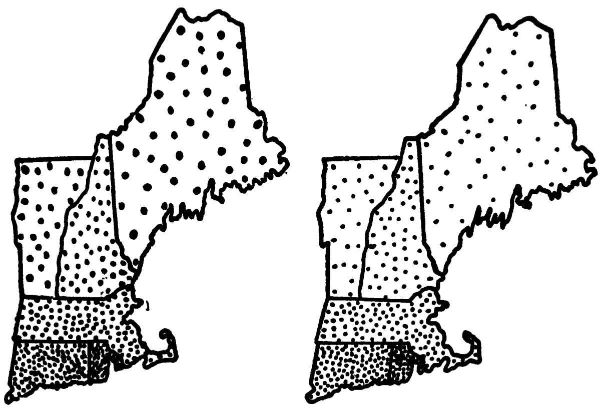
EXHIBIT 197.—Use of dots in indicating density

contrary, we are interested in relative concentration, the most satisfactory map is the density map, which will now be considered.

Densities.—In studying most area distributions, the important questions are: Where are the points showing a high degree of concentration? and Where are the sparse distributions? In showing distributions where area is a factor, the density map made with dots, as illustrated in Exhibit 197, is commonly used. In this type of map all dots are of the same size, and the distributions are shown by the frequency with which these dots occur. Numbers are not well shown by this kind of map but the relative densities are clearly indicated. Such a value should be assigned to the dot as will best fit in between the

extremes. That is, on the one hand, the value should not be so small that there would not be room for the dots in the concentrated areas—they may run together, but there should not be several layers—while, on the other hand, the unit should not be so large that important data in the sparse areas would be incorrectly represented or omitted.

The dots representing data should be spread evenly over the corresponding units of area.[1] Care should be taken to see



A

Exhibit 198.—*A*, Wrong because the larger dots in the sparse areas exaggerate the data. *B*, Right because density is correctly shown by dots of equal size.

that the dots are not placed in rows on a density map. It is important to use dots of uniform size. Otherwise the effect will be misleading as illustrated in Exhibit 198*A*.

Internal boundary lines, such as state lines on a map of the United States, are sometimes omitted as in Exhibit 197, but they are usually left in as a guide to the reader. In Exhibit 199, which is plotted on a county basis, state lines are shown but county lines are omitted.

Since the areas of the states on the map are proportional to their actual areas, the dots are automatically distributed on a

[1] After the unit has been selected, the number of dots for each state or division may be determined and the figures penciled on each area. Then, with a single stroke of the pen for each dot, and counting them as you go, keep working over the whole area, placing a dot where the largest space remains until the required number is placed. The Payzant pen is good for this work since it makes a perfectly round dot.

square-mile basis if they are spread evenly over each state or other unit area. Maps like Exhibit 197, of course, show the average distribution for each state, and so do not show the concentration points at cities.

The concentration points show up best when a map is made on the basis of small divisions, such as counties on a map of the United States (Exhibit 199), or townships on a map of a state, or blocks on a map of a city. The degree of accuracy necessary for maps of this kind depends upon the purpose for which the map is to be used. In Exhibit 200 each dot is carefully located to

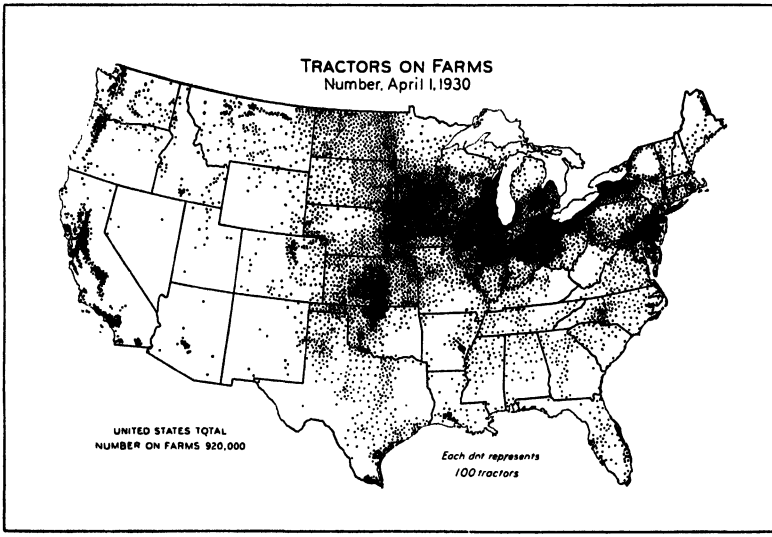


Exhibit 199.—Farm tractor density map of the United States drawn on a county basis. (*Courtesy of United States Department of Agriculture.*)

represent 200 persons. The dots here are not spread evenly over any particular area but are located near the center of gravity of each group of 200 persons.

The type of map illustrated in Exhibits 197, 199, and 200 gives the impression of gradual changes from dense to sparse distributions, and seems to smooth out the breaks between political jurisdictions, which are especially noticeable when shading is used as in Exhibit 201. Ordinarily it is not necessary to use the cross-hatching method in maps like Exhibit 201 when area is a factor. The actual areas are proportional to the areas on the map, and the density can be appreciated better by using a dot

map like Exhibit 199. In Exhibit 201 the class intervals vary in size from two in the lowest group to forty-five in the next to the highest group, with no upper limit upon the highest group. As

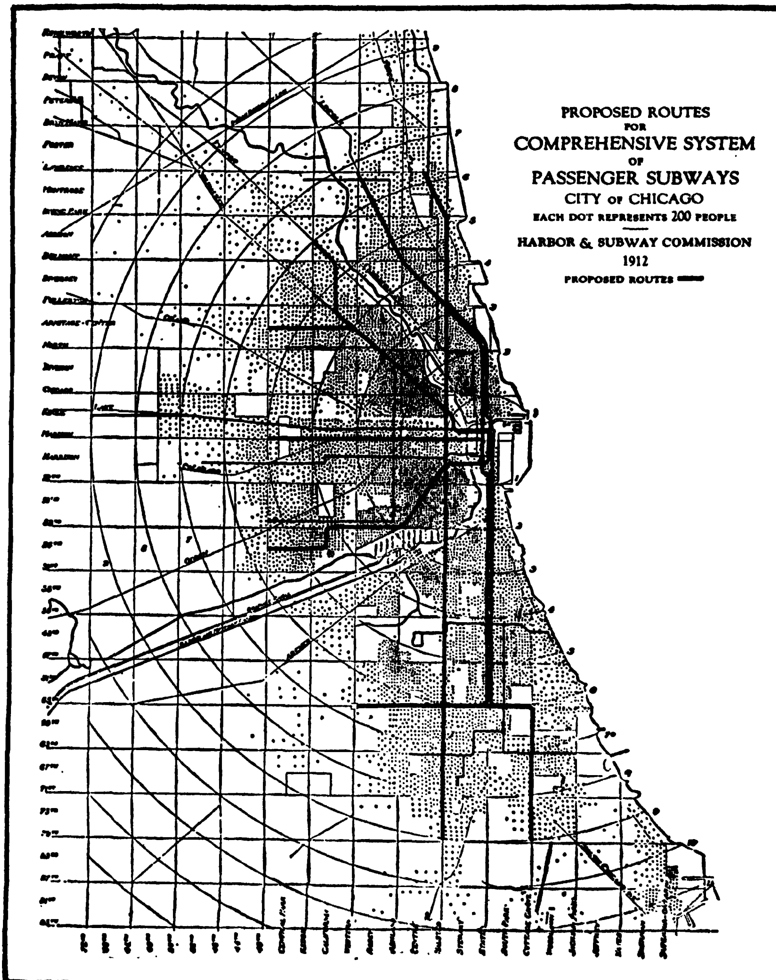


Exhibit 200.—Density map with dots representing 200 persons on a center of gravity basis. (*Courtesy of Department of Public Works, City of Chicago.*)

a rule, class intervals should be equal in size on maps of this kind, though occasionally there may be practical exceptions to this rule. The difficulty, of course, is to show the sparsely populated

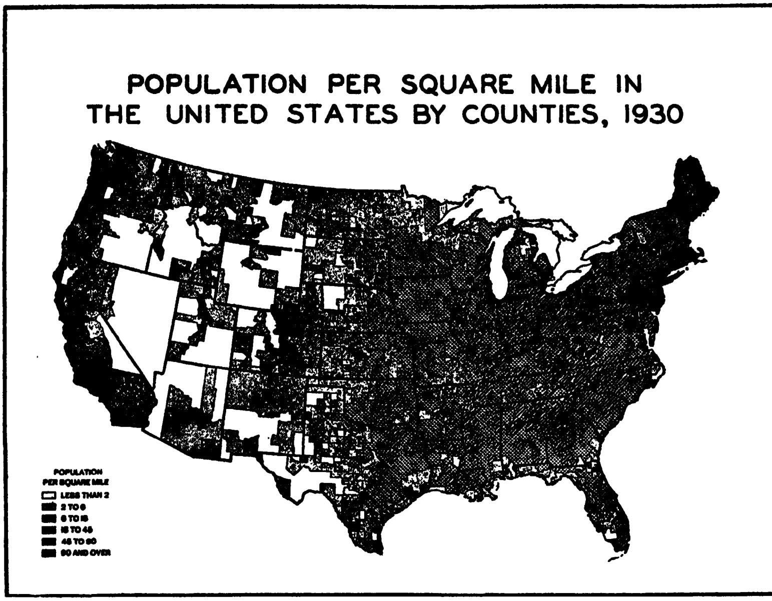


EXHIBIT 201.—Use of shading to show density. In showing density, dots are ordinarily preferred to shading. (*Courtesy of United States Bureau of the Census.*)

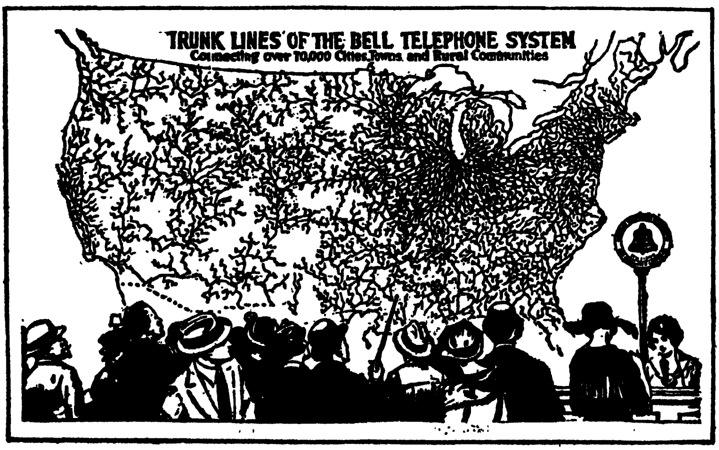


EXHIBIT 202.—Density of linear magnitudes. (*Courtesy of American Telephone and Telegraph Co.*)

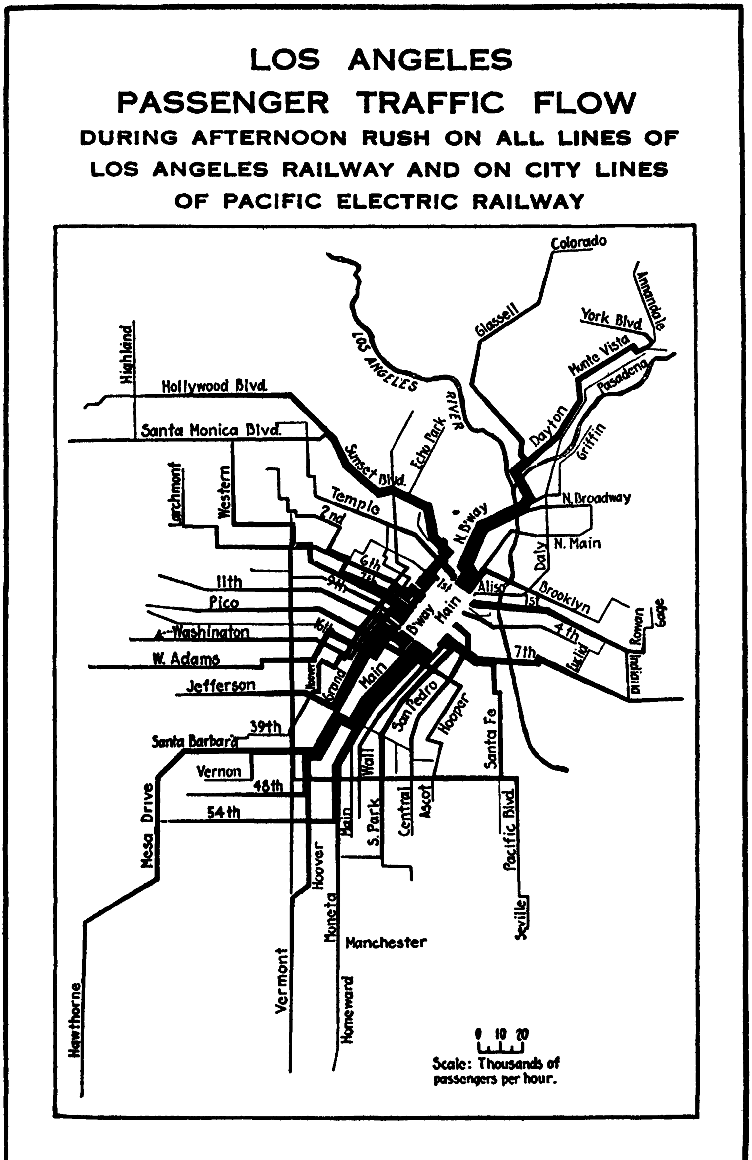


EXHIBIT 203.—A traffic flow map. (*Courtesy of Joe R. Ong.*)

areas accurately. This would be much easier to do by using a dot density map such as Exhibit 199.

While dots are very satisfactory for representing separate objects, such as automobiles, factories, and persons, they are not truly representative of the density of linear magnitudes, such as roads, railways, and telephone lines. This kind of density distribution is very clearly shown by using a map like Exhibit 202 which shows the density of Bell Telephone trunk lines simply by mapping the lines in their actual positions.

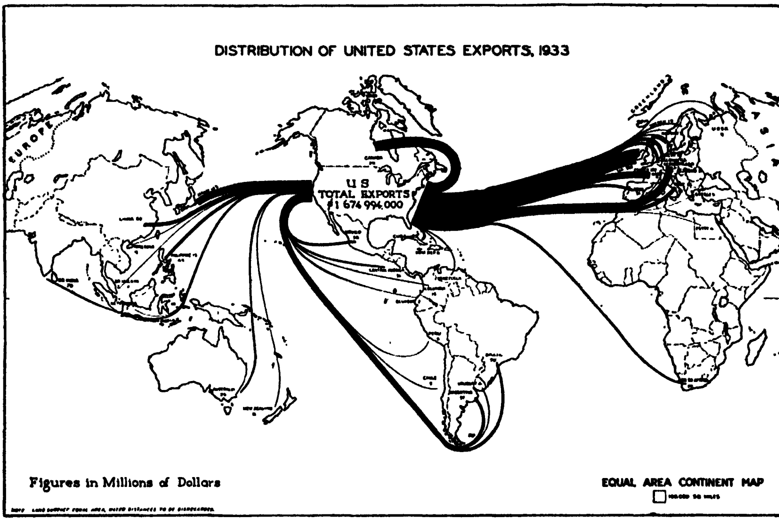


EXHIBIT 204.—World flow of United States exports. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

Flow Maps.—A special type of density map is the flow map, which shows the density of such flows as those of traffic (Exhibit 203) or merchandise distribution (Exhibits 204, 205, and 206).

In Exhibit 203 the density of street railway traffic is shown by the widths of the lines. This is the common method of showing traffic flow. The widths of the lines are laid out to a common scale, such as thousands of persons per hour, which should be made large enough to show all important traffic, and yet it should be small enough so that there is room for the lines in the congested districts.

In Exhibit 204 the same method as that of Exhibit 203 is used to show the world distribution of United States exports, the

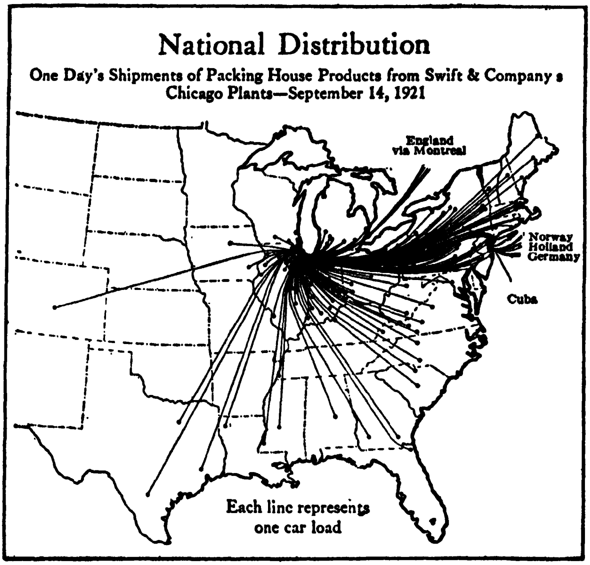


EXHIBIT 205.—Density of traffic flow. (*Courtesy of Commercial Research Department, Swift & Co.*)

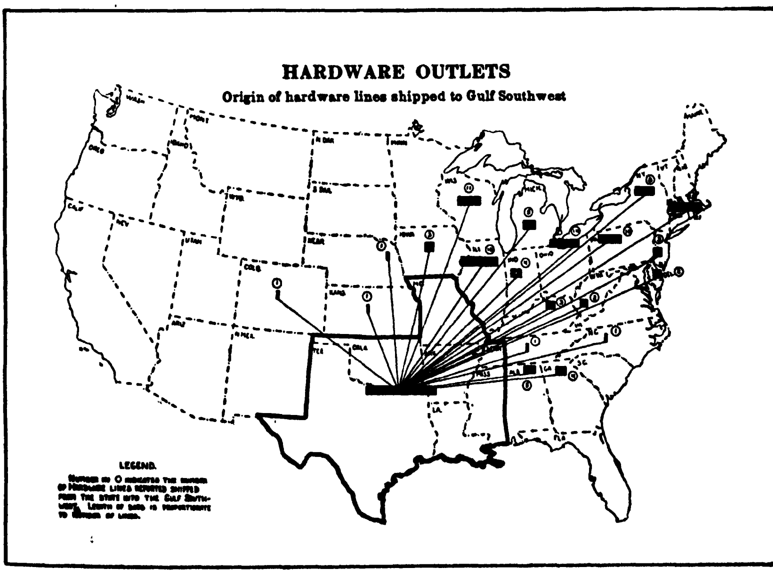


EXHIBIT 206.—Merchandise flow map. (*Courtesy of United States Bureau of Foreign and Domestic Commerce.*)

widths of the flow lines indicating the different amounts of commodities exported.

The concentration area of Swift & Company's market is shown by Exhibit 205 which is designed to show the density of the flow lines rather than the importance of each flow. A similar problem is illustrated in Exhibit 206, which shows the importance of each line of flow by means of bars at both source and destination.

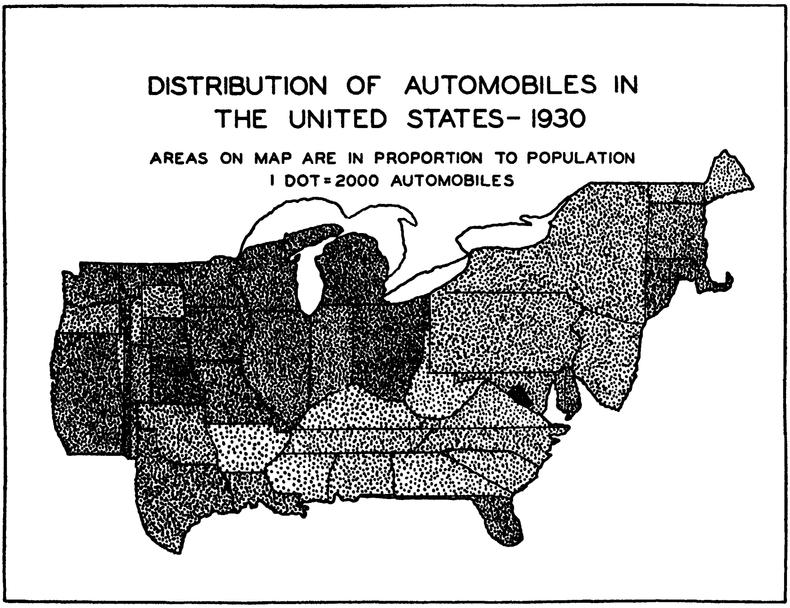


EXHIBIT 207.—Showing the relation of population to automobiles by drawing the states in proportion to population. (Not recommended for general use.)

Geographical Distributions Where Something Other than Area Is a Factor.—When a distribution is to be shown where something other than area is a factor, such as number of automobiles per thousand of population, the density dot map cannot be used in the form just described. A method sometimes used is illustrated in Exhibit 207, in which the states are drawn, not in proportion to their actual areas, but in proportion to their populations. Then the dots representing automobiles show their distribution in relation to population. As previously mentioned, however, the method of distorting the map to represent something other than area is not recommended for general use, as it is quite

likely to confuse the reader and leave an incorrect impression of sizes, distances, and relative positions of the areas represented.

A better form for showing the data of Exhibit 207 is the cross-hatched map illustrated by Exhibit 208. This is not an absolute rule, however, since the two maps do not show quite the same thing. Exhibit 207 compares populations of states and shows the relative density of the automobile-owning population but it does

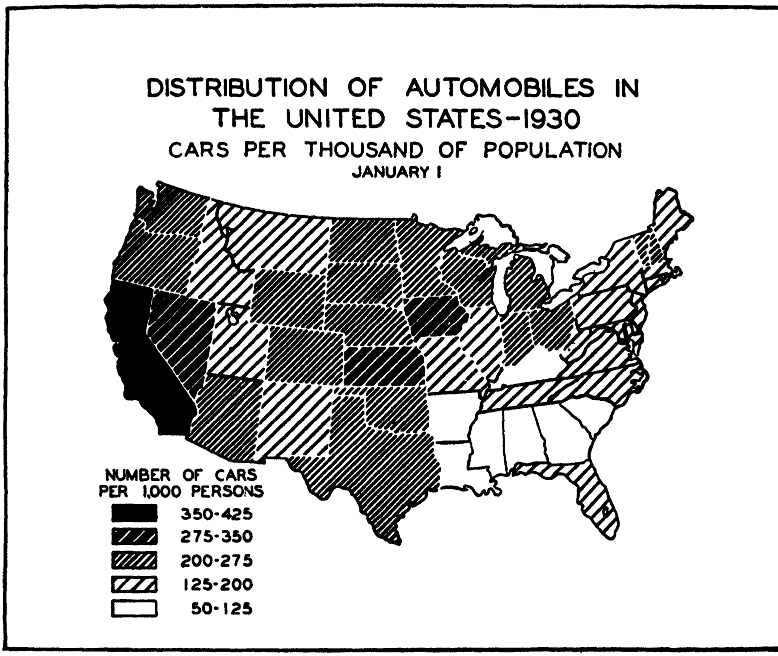


EXHIBIT 208.—Showing the relation of population to automobiles by a simple method of shading. Usually this method is to be preferred over the method shown in Exhibit 207.

not show area, while Exhibit 208 compares areas but does not show comparative populations. The range of shading in Exhibit 208 is graduated equally from white to black in five shades. The middle shade is half black and half white. The second one from the top is the negative of the second from the bottom. The shades vary by 25 per cent from one extreme to the other. That is, the bottom shade is 0 per cent black, the second is 25, the third is 50, the fourth is 75, and the top is 100 per cent black.[1]

[1] Ideally, of course, 100 per cent black would represent the saturation point. In this map it is used merely to represent the highest class.

In making map shades, since the differences in the scale are represented by differences in the proportion of area covered by ink, it is best to make the differences as simple as possible. An elaborate design of shading cannot be understood so easily as

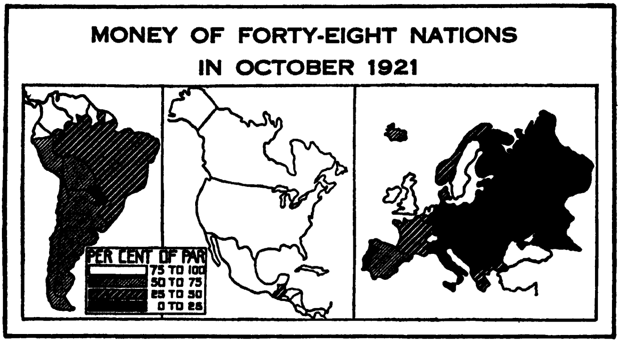


EXHIBIT 209.—A simple method of shading to show percentages. (*Courtesy of Cleveland Trust Co.*)

the simple forms illustrated in Exhibits 208 and 209, in which even the direction of the lines does not change. Any number of shades can be made very easily, and with their differences mathematically exact, by using the method of cross-hatching

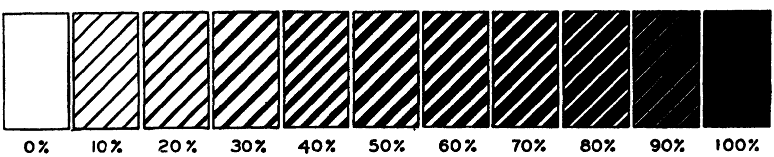


EXHIBIT 210.—A simple method of shading mathematically graduated from 0 to 100.

illustrated in Exhibit 210. In this exhibit the shades are shown, with variations of 10 per cent, from zero to one hundred.[1]

[1] This shading is very simply and quickly made with the aid of a section liner. (This instrument is described briefly on p. 231.) Simply determine the width of line to be used and adjust the section liner so that it will move exactly this width. Then, in making a 10 per cent shade, for instance, draw a line, then move the section liner nine times for the white part, and then move it again for another black line, and so on. For a 50 per cent shading draw five lines together, and then move the section liner arm five spaces for the white part, and so on. For a 90 per cent shading, draw nine lines together and skip one, and so on.

If a large amount of map work is to be done for reproduction by a photo-

Usually it is not practical to have as many shades as are shown in Exhibit 210, but it will be noted that the values represented can be very quickly appreciated even without reference to the scale. It is not difficult to see, for instance, that 10 per cent of the area is black in the 10 per cent shade, or that half of the area is black and half of it is white in the 50 per cent shade.

Such maps as Exhibits 208 and 209 give the impression that absolute uniformity prevails within each of the divisions and that changes occur only at their borders. While this would be true, practically speaking, of Exhibit 209, it would not be true of Exhibit 208 where the changes naturally occur gradually from one district to another. In this respect the distorted map of Exhibit 207 has the advantage. In instances where abrupt changes actually occur, and where conditions within the areas are practically uniform, the cross-hatched map leaves correct impressions of the distribution within each area. In instances where the distribution within each district is not uniform, the reader usually will make the proper allowances if he appreciates that the figure represented really is an average.

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graphic process, a great amount of labor can be saved by drawing the shading on sheets and making photographic copies. If the photostat method is used, only the lower half of a series of shades, like Exhibit 210, need be drawn, for negatives of these will make positives of the upper half. These sheets are cut and pasted on the map in the proper positions.

In mounting such shades as the foregoing, only the edges adjoining the finished part need be trimmed to fit. Thus, assume that we are pasting shades on a map to make Exhibit 208. Suppose that we start with the state of Washington. We will cut the north and west sides to fit and let any surplus project over Idaho and Oregon. Then, when we mount the shade for Oregon, we shall trim the west and north sides properly and mount it in its proper position, covering the surplus shade of Washington and allowing the surplus shade of Oregon to extend over the unshaded states to the south and east. Then, when we mount the shade of Idaho with its western side correctly shaped, it will cover the surplus of Washington and Oregon and give their eastern boundaries their correct shapes. This process can be carried on until the last states are reached when the shades will have to be trimmed all around before mounting. Or, if one prefers to do so, the shades may be allowed to extend over the national borders, and, when completed, the whole map may be trimmed and remounted.

For the sake of simplicity all shading lines should be made in the same direction. The lines should be drawn at an angle of about 45 degrees with the border line of the map in order that the changes may not be noticed in the angles that these lines form with the meridians and parallels of latitude.

Exhibit 211 represents a percentage map on which a somewhat more complicated system of cross-hatching is used than that represented in Exhibits 208, 209, and 210. Methods of cross-hatching similar to that in Exhibit 211 are widely used and are designed to show the differences distinctly. The statistical advantage, however, rests with the method illustrated in Exhibit 210. If the method of Exhibit 210 were used in Exhibit 211, only the shades from the lower part of the scale would be used since there are no data to be represented by the upper part of the scale.

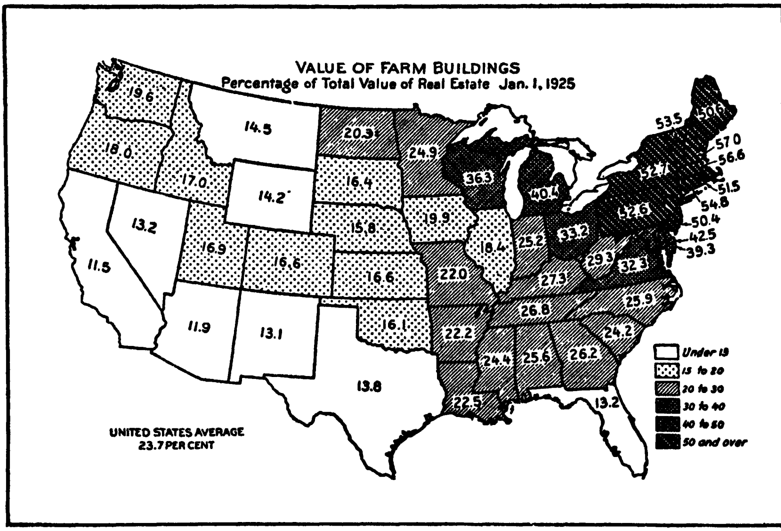

Exhibit 211.—Shading to show percentages. The general type of cross-hatching illustrated is widely used. Compare with Exhibits 208, 209, and 210. (*Courtesy of United States Department of Agriculture.*)

Use of Colors in Statistical Mapping.—The use of colors in statistical map work is limited. No scheme of representing different values by different colors can be used to give a visual impression that corresponds directly with the distribution of the data. No arrangement of red, orange, yellow, green, blue, and violet can give a clear impression of a graduated series of numbers. That is, for instance, green would not naturally impress a reader as representing a larger or smaller number than blue or red. Even if different intensities of the same color, graduated from light to dark, are used, the maps are more difficult to interpret than they are when the cross-hatching method is correctly used, and reproduction is much more expensive.

Colors are very useful in distinguishing one area from another or one series from another, but, as a rule, they should not be used to represent values. Another effective use may be made

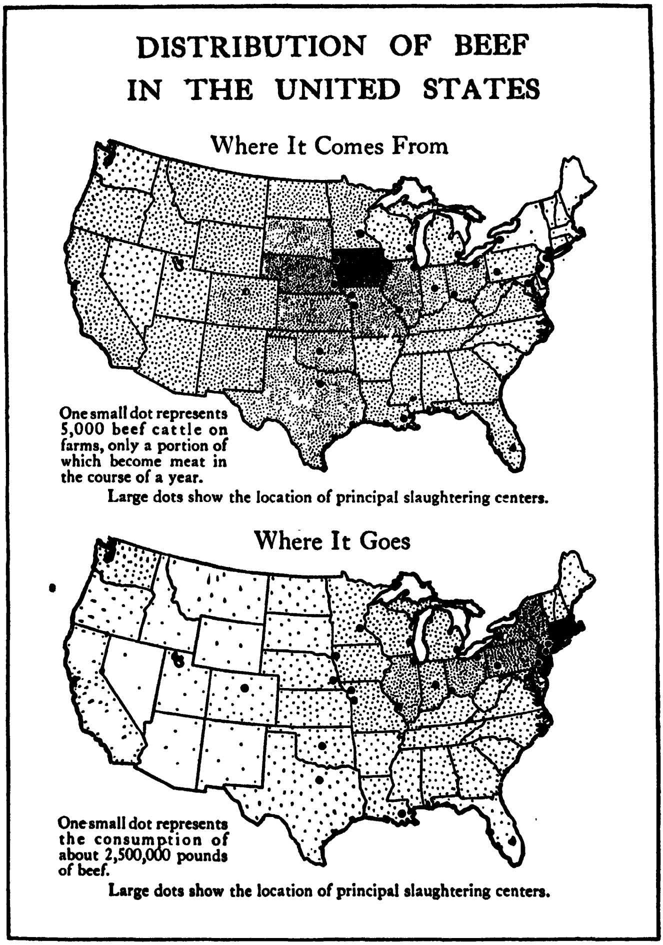

EXHIBIT 212.—Comparing geographical distributions. (*Courtesy of Commercial Research Department, Swift & Co.*)

of colors for showing positive and negative quantities. For instance, profits may be shown in black and losses in red.

Comparisons of Geographical Distributions.—When it is desired to compare different geographical distributions for the same area, it is usually best to make different maps of the data

as shown in Exhibits 212 and 213. In such cases the maps should be drawn to the same scale and they should be plotted in the same units if possible. Because of the different units used in Exhibit 212—number of cattle on one map and pounds of beef on the other—the comparison is more difficult to comprehend than it would have been had the dot been given the same value on both maps.

Exhibit 213 illustrates a good method of showing the changes in the sources of Boston's milk by showing the actual distribution

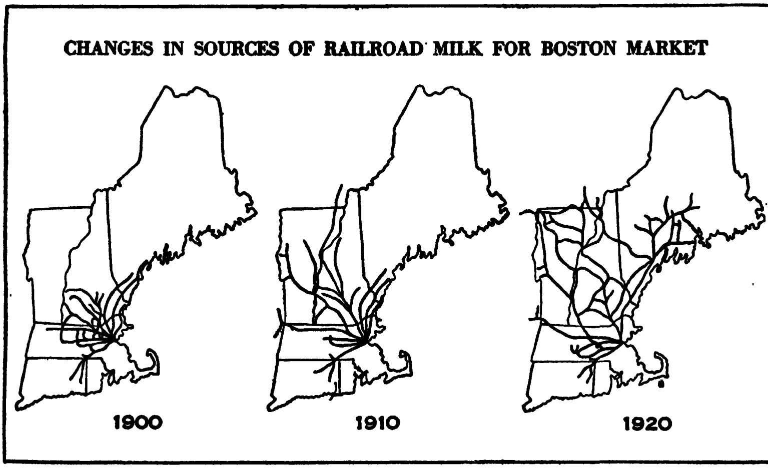

EXHIBIT 213.—Comparing geographical distributions. (*Courtesy of Boston Chamber of Commerce.*)

of the routes over which it is carried. In this chart no attempt is made to show the importance of the various sources.

The Map-tack System.—There are many mechanical aids on the market that are useful in graphically showing geographical distributions. Chief among these are the mounted maps, tacks, pins, beads, cords, rings, tags, and flags, which are used together in various combinations in what is commonly known as the map-tack system.[1]

[1] Complete map-tack systems may be obtained from Rand McNally, Chicago; National Map Co., Indianapolis; and Educational Exhibition Co., Providence. Inexpensive spherical headed pins may be obtained at 10-cent stores. If suitable colors cannot be obtained, the pin heads can be dipped in paints or enamels of the desired shades.

The map-tack system is a valuable aid in showing the routing of salesmen; in designating towns, branches, and agents; in classifying purchases; and in analyzing competitors' strengths. This system of map making is very flexible. Tacks or other marking devices may be added, changed, or removed readily according to current requirements.

If tack maps are made for photographic reproduction, or for those to read who are not in close touch with the working of the particular map system used by the individual office, they should

EXHIBIT 214.—A large-scale wall map. (*Courtesy of Rand McNally & Co.*)

be simple in design and made according to the dot map principles discussed previously in this chapter.

If, however, the map is made to be used by those who are in close touch with the individual system, it may be more complicated, though care should be taken to keep the system simple enough to be understood easily by those who are to use it. Disassociated facts may be successfully represented on the same map by different devices or different colors if the readers are in close enough contact with the system to become familiar with their meanings quickly.

Tack maps vary in size from small hand maps to huge wall maps such as Exhibit 214. These large maps sometimes are

mounted in sections. It is important to have a map large enough, especially if there are congested districts to be studied. It is better to have a map too large than too small.

Map Mount.—Tack maps must be mounted upon a kind of board that will allow the pins to be pushed in to their heads without difficulty and yet hold each one securely. Unless the pin is pushed in to its head, it is easily knocked off the map, and as each pin is usually placed in position currently as the data are obtained, sometimes with no tabular record, it cannot be replaced without great inconvenience.

Wood makes a satisfactory mounting for use with tacks if it is very soft, and made up, say, three-ply to prevent warping and

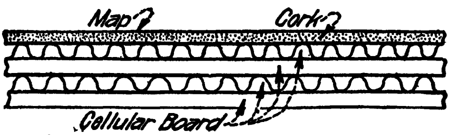

EXHIBIT 215.—Cross-section of tack-map mount. (*Courtesy of Educational Exhibition Co.*)

cracking. A very cheap yet satisfactory mounting can be made with three or more layers of corrugated strawboard.[1] If the tacks are to be removed often, the strawboard should be faced with cloth or, better still, with cork. The structure of a form of ready-made mount which can be purchased is illustrated in Exhibit 215. This mount consists of a cellular strawboard built up several ply, the ribs in the alternate layers running crosswise to those in the other layers. On the surface of the strawboard is placed a thin layer of cork on which the map is directly mounted. While it is easy to push a pin into this kind of board, there is enough frictional resistance to hold it securely, and moreover the resilience of the cork closes the hole so that tacks may be pushed in many times at the same place without danger of working loose. For temporary work, especially if the pins are not to be removed,

[1] These layers should be glued together with the ribs in the middle layer running crosswise to the others. The map should be mounted with paste. In order to equalize the shrinkage and tendency to warp, a piece of plain paper should be pasted on the back of the board at the same time the map is mounted. The finished map should be dried in a press or under weights. If the map is not printed in fast colors, it should be covered with a fixative sprayed on with an atomizer before being wet with paste.

a single thickness of corrugated strawboard, mounted on a frame so that the tacks or pins may project through the board, has been found to be quite satisfactory.

It is desirable to size and varnish the faces of tack maps as a protection against dampness. This treatment also makes it possible to wash maps that become soiled.

Unless tack maps are very small, they should be provided with some kind of a substantial frame. This is especially necessary to prevent warping. Examples of the common forms of protection are the map cabinet (Exhibit 216), the wall frame (Exhibit 219), and the swinging display fixture (Exhibit 223, page 214).

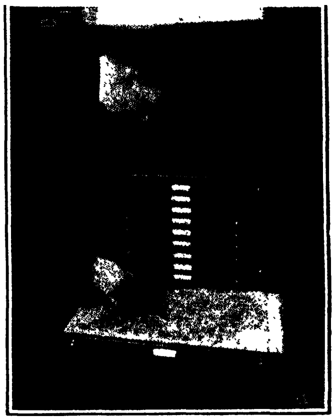

EXHIBIT 216.—A map cabinet. (*Courtesy of Rand McNally & Co.*)

Map Marking Devices.—Of the devices used to indicate facts in the map-tack system, the most common are tacks, pins, beads, rings, flags, tags, stickers, and cords. The most important of these devices is the tack or pin.

Map tacks or pins are made with sharp needle points, and with glass, celluloid, and cloth-covered heads which are made in several hundred variations of sizes and colors. Cloth-covered tacks soil and fade more easily than do the glass and celluloid tacks. Exhibit 217 illustrates the use of spherical glass-headed tacks and flat celluloid tacks for representing automobile sales data. Such a use of maps and tacks presents a picture of the distribution that is practically impossible with tables. The large celluloid tacks are surfaced so that numbers or other designations can be written upon them. Different distributions can be presented clearly by using different-colored tacks, but different colors should not be used for presenting different degrees of value in the same distribution. For instance, one might show clearly the distribution of population with black tacks all of the same size, and the sale of automobiles by using white tacks all of the same size, but from a graphic point of view it is not good practice to make a map allowing, say, one black dot to represent one

automobile and one white dot ten automobiles. In this respect the map illustrated in Exhibit 217 is faulty. The tack representing the sale of ten cars does not directly impress one as being ten times as large or important as the tack representing one sale.

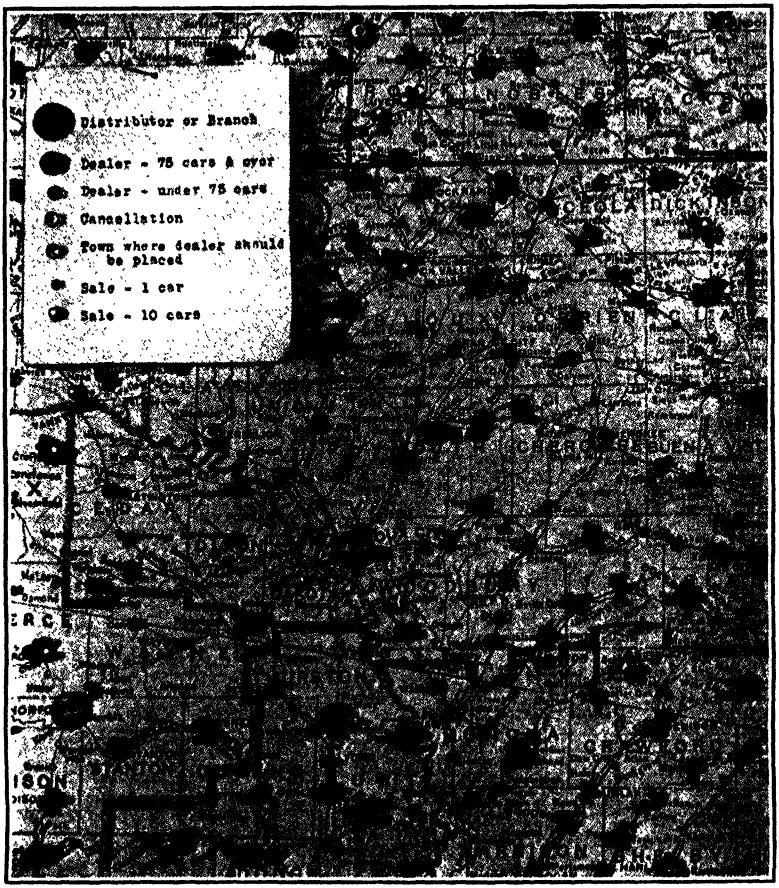

EXHIBIT 217.—A tack map formerly used by the Willys-Overland Company. (*Courtesy of Rand McNally & Co.*)

Looking over the area as a whole, it is difficult to pick out the areas of greatest density of sales because the visual impression left by observing, for instance, two of the large white-topped tacks is much weaker than that left by observing twenty small black tacks. From a graphic point of view, the visual impression should correspond directly with the facts presented. Other-

wise, very inaccurate impressions are likely to be gained. Thus, in Exhibit 217 the map would be improved by using ten small pins instead of one of the larger white-topped pins. There would be no difficulty in making this change on the area shown in Exhibit 217, but in areas of great density there would not be room for so many pins if the plotting were on the basis of one pin for each sale.

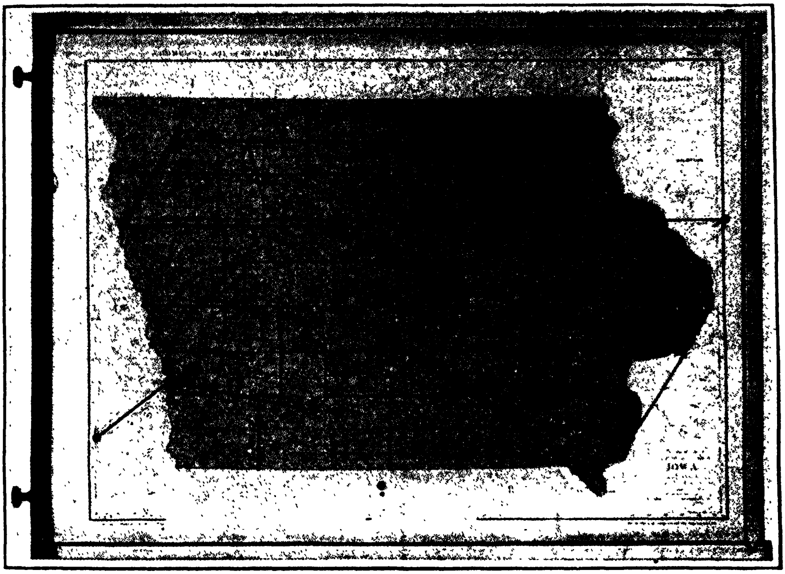

EXHIBIT 218.—Use of map cord. (*Courtesy of Rand McNally & Co.*)

A method of showing more accurately distributions which vary from extreme sparseness to great density is the use of beads and pins. By this method, the spheres are not spread over the area but are piled up perpendicularly from the map, making columns similar to those of Exhibit 189 on page 178. Where only one dot appears, a map tack is used. The larger values are represented with long pins strung with beads which are exactly like the heads of the tacks. If the long pins available for this purpose are too short, steel wire can be used. When a large number of beads are used on a pin or wire, it is well to make every tenth bead another color so that it will be easy to read the number.

For supplementing map tacks and beads, other devices, such as rings, tags, flags, and stickers, are sometimes used to show addi-

tional information. These may bear notations or they may represent the information by their colors or forms.

When territories are to be marked off or routes are to be laid out, the lines can be made conveniently by using map cord as

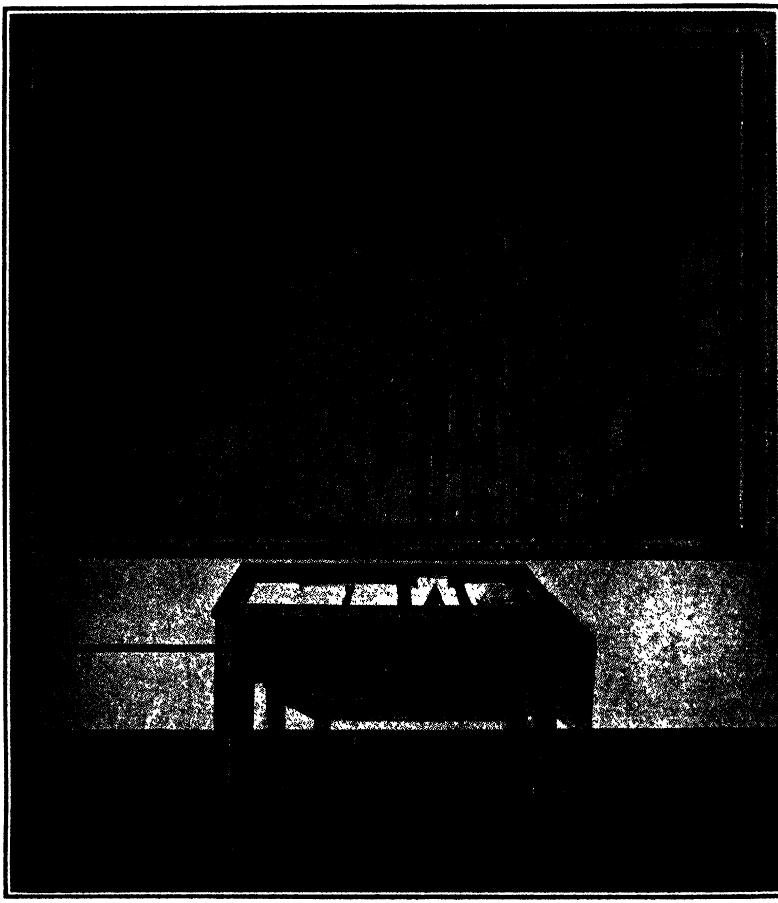

EXHIBIT 219.—International Harvester Company's tack map with card index system. (*Courtesy of Rand McNally & Co.*)

shown in Exhibit 218. Any desired changes can be made very easily and without damage to the map simply by shifting the cord.

Washable maps can be secured which will allow marking and erasures. They may be marked with pencil, crayon, or pen,

and erasures are made by washing as one would wash a blackboard or slate.

Often it is desirable to have some means of tying the information of a map to the more complete and detailed information given in the various records of the firm. Exhibit 219 illustrates a method of using a card index system in connection with a pin map.

CHAPTER IX

DUPLICATION, REPRODUCTION, AND DISPLAY

Since it is usually necessary to have more than one copy of a chart, it is important to give some consideration to methods of duplication and reproduction. The choice of the particular method to be used will depend largely upon the purpose of the chart. If, for instance, only a few copies are to be made for the use of a board of directors, the method may differ greatly from the one used to reach the public at large through a magazine.

Blueprinting.—The method most commonly used by engineers in duplicating drawings is that of blueprinting. This method is the cheapest one readily available for making a small number of direct copies. A drawing intended for blueprinting is usually laid out on drawing paper and then traced in ink upon tracing paper or tracing cloth which is translucent. Tracing cloth is usually preferred to tracing paper because it is more satisfactory to work upon, more translucent, and more durable. If the drawing is not too complicated, it may be made directly upon the tracing cloth or paper.[1]

In making a drawing on tracing cloth, the surface of the cloth should be rubbed with powdered chalk or pounce to remove the traces of grease so that the ink will flow freely and evenly. To insure a good line, an opaque ink must be used. Colored inks often are not heavy enough to print well. To produce a light shading, thin colors or pencil tints are sometimes spread over an area on a tracing instead of cross-hatching. Such an area will print in very light blue when a blue print is made.

Working charts and preliminary drawings are sometimes blueprinted from bond or ledger paper, but such prints do not always present a pleasing contrast and the watermarks show plainly. Any clear, white paper, however, may be made translucent enough to make a good blueprint by oiling with "Nujol"

[1] See pp. 218 and 219 for a further description of tracing cloth and paper.

or by applying "transparentizing solutions," which are handled by most dealers in drawing supplies.

If typewritten material is to be blueprinted, the copy should be made on thin paper which is not watermarked, a good ribbon should be used, and a good carbon paper should be placed in the machine to face the back of the paper on which the material is being typed. With the ribbon ink on one side of the paper and the carbon ink on the other, a fairly opaque letter is made.

Blueprinting, after the original is made, is a simple process. A piece of sensitized blueprint paper, with the tracing in front of it, is exposed to the sun or electric light for a few seconds or minutes, depending upon the rapidity of the paper. A chemical action takes place in the exposed parts which renders the coating insoluble. This turns blue when the paper is washed in a water bath, and the soluble parts, which were protected from the light by the ink on the tracing, wash out leaving white lines. That is, the ordinary blueprint reverses the black and white of the original and the black lines on the tracing are white on the blueprint. On an overexposed print, the blue color may be intensified and the white lines cleared by dipping in a bath containing potassium bichromate or sodium bichromate (1 or 2 ounces of crystals per gallon of water) and again washing. Blueprint paper may be bought in various widths, weights, and speeds of printing. When fresh, it is of a greenish-yellow color. It deteriorates rapidly and cannot be used at all after a comparatively short time.

Several blue-line and black-line papers are available with which positive copies can be made directly from a tracing. Another method of duplicating positive prints requires that a negative first be made on Van Dyke paper,[1] which is a thin paper that turns dark brown on exposing and fixing. Then from these negatives blue-line prints with white backgrounds can be made from blueprint paper, or from blue-line print paper, and brown-line prints can be made from a special paper prepared for the purpose.

The equipment used in blueprinting varies all the way from almost nothing to the type of machine illustrated in Exhibit 220. In this machine the tracings and paper are fed through rolls and

[1] This paper is given different names by different companies.

the printing, washing, and drying are done in one operation. As electric light is used, the operator is independent of the sunlight. Frames mounted on carriers so that they may be run out of a window are widely used when the exposure is made to the sun. Good prints may be made from small tracings simply by holding them to the sunlight against a window pane.

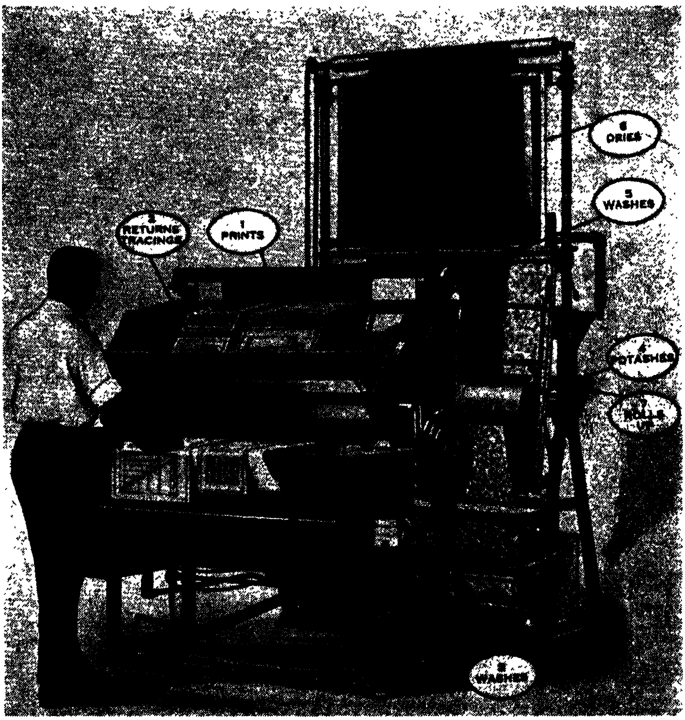

Exhibit 220.—Electric blueprinting machine with washing and drying equipment. (*Courtesy of The C. F. Pease Co.*)

By keeping a small quantity of blueprint paper on hand, it often happens that copies of tables, graphs, or printed matter may be made in this simple way much more quickly and cheaply than they can be made by hand or on a typewriter.

If an exact duplicate of a tracing is wanted, there are firms that do this kind of work by a gelatine process in which a special matrix print is made in a blueprint outfit and transferred to a gelatine-surfaced table. The impression is then inked and prints

are pulled from it which can be used for making blueprints equal in every respect to those made from original tracings.[1]

Good photostat negatives oiled with "Nujol" or any purified paraffine oil, and well blotted, make satisfactory negatives for blueprinting.

Unless one regularly has a large amount of blueprinting work, it is usually sent out to commercial blueprinting houses. When the amounts of work are small or irregular, this method is ordinarily cheaper and more convenient than operating and maintaining blueprint equipment.

Drawing for Photographic Reproduction.—Much reproduction work is done by methods which include some use of photographic processes. This kind of work includes that of the photostat and the many kinds of plates or "cuts" for printing the illustrations in books, magazines, newspapers, circulars, and other publications.

Drawings for photographic reproduction should be made on smooth white paper or tracing cloth and in black ink for the best results. Green, red, and yellow inks will photograph (as black) but pure blue will not show at all in ordinary light. Usually a chart that is to be reproduced by a photographic process is made somewhat larger than the reproduction. The reduction tends to eliminate minor irregularities and gives a much more finished appearance than is possible when charts are reproduced in the exact size of the original. Exhibit 221 shows how a full-sized reproduction looks when it is reduced one-fourth and one-half linear. Drawings are sometimes made many times the size of the reproduction, but as a rule originals made from two to four times (linear) the size of the reproductions will give the best results.

In drawing for reproduction, particular attention should be given to the widths of lines. A three-fourths reduction (linear), of course, reduces the width of the line to one-fourth of its original width. A very helpful aid in judging the appearance of the reduction is the reducing glass, which is a concave lens mounted like a reading glass.

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[1] One company which duplicates tracings is the Lithoprint Company of New York, 41 Warren St., New York City.

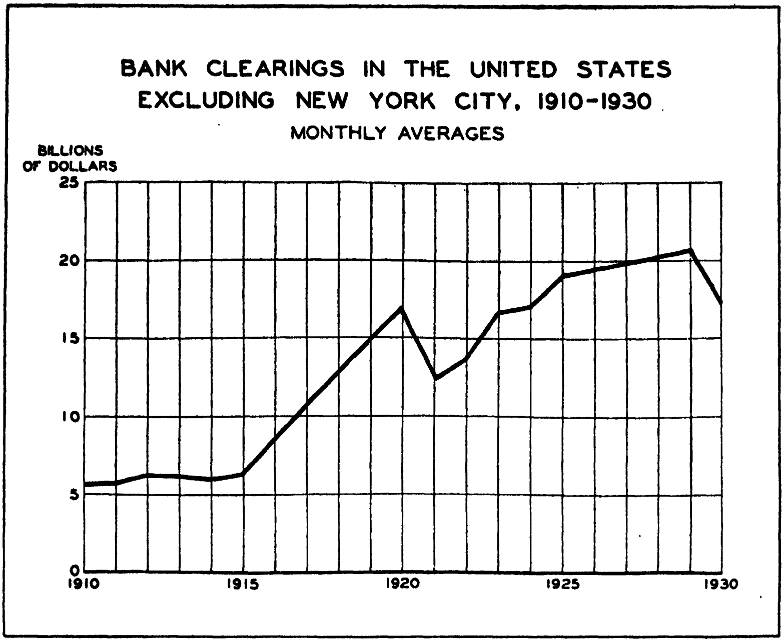

EXHIBIT 221*A*.—Drawing—full size.

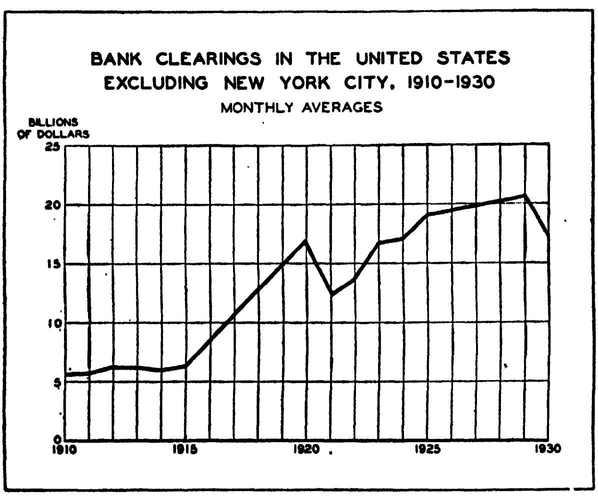

EXHIBIT 221*B*.—One-fourth reduction of Exhibit 221*A*.

On drawings for photographic reproduction, irregularities and errors can be corrected by painting out with Chinese white (water color). Lettering can be printed with type on strips of paper and these strips can then be pasted in their proper positions on the drawing. If the edges are carefully painted with Chinese white, and care is used in photostating, the edges of the pieces pasted on will not show. Or if a line cut is made, any such lines will be tooled out by the engraver when the cut is finished.

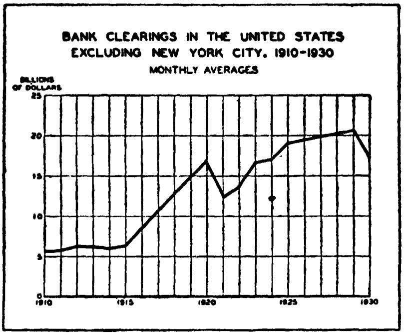

EXHIBIT 221*C*.—One-half reduction of Exhibit 221*A*.

The Photostat.—There are several direct photographic methods for reproducing drawings but the most commonly used is the photostat (Exhibit 222). In the photostat process the sensitized paper is exposed to the chart through certain lenses and mirrors, so that by a direct exposure a positive print of white lines on a black background can be made. This method is especially useful for copying charts or printed matter from reports, books, or magazines; and when only a few copies of a chart are to be made, it is quite economical. Many firms which have a large amount of copying to do have their own outfits, while others find it best to have the work done by commercial photostating firms.

Cuts or Plates for the Press.[1]—When charts are reproduced by the printing press, plates or "cuts" are made from the originals by various photomechanical processes.

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[1] For a good discussion of this subject see Herbert W. Smith, "Picture Plates for the Press," *University of Missouri Bulletin*, Vol. 22, No. 28.

Statistical charts are usually black-and-white copy with no gray tones and are commonly reproduced by cuts known as *line plates* or *line drawings.* These cuts are made by the process known as zinc etching, in which the drawing is photographed and a positive is printed on a sensitized zinc plate which is etched (corroded) with acid, leaving the protected parts or lines in relief. This is the cheapest and most generally used method of reproducing charts for the press. Care should be taken to see that the drawings are well made since all imperfections are, of course, reproduced.

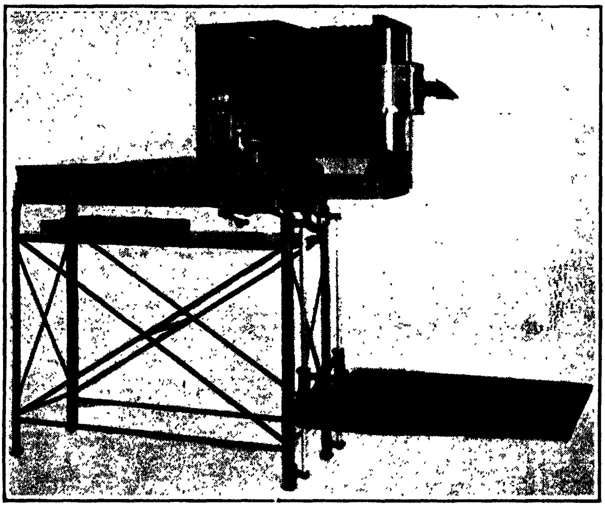

EXHIBIT 222.—Number 1 photostat. (*Courtesy of The Photostat Corporation.*)

A method of making cuts, which does not require a carefully drawn original, is that of wax engraving. In this process the engraver does the work of the draftsman and the lettering is put in with type. Thus, a very good cut can be made from a rough pen or pencil sketch. The chief disadvantage in using wax cuts at the present time is the relatively high cost of engraving them.

Another kind of cut, occasionally used in connection with graphic presentation, is made by the half-tone process, in which a negative is made through a screen which breaks up the shades into series of dots of different sizes. The eye blends the black tone of the dots with the white background of the paper and gives an impression of a gray tone. For different kinds of paper,

screens of different fineness are used, varying from sixty lines to the inch for rotary press newspaper work to one hundred and fifty lines to the inch for the finest enamel papers. The most satisfactory half-tone illustrations are made from photographs or wash drawings. When photographs are used, they should be made on glossy paper to produce the best results.[1]

Ben Day Shading.—The high cost of color printing prohibits its use for most purposes. When areas must be contrasted, usually some form of black-and-white shading or cross-hatching is used. This may be drawn by hand or it may be produced mechanically by the Ben Day process. The Ben Day shade is printed mechanically either on the plate by the engraver or on the original drawing from an inked gelatine surface and rubbed on with a stylus. Most good engravers can do Ben Day work. The particular shades to be used are selected by the maker of the chart from a catalogue and designated by numbers on the drawing. An example of a chart on which Ben Day work is used is shown in Exhibit 121 on page 103.

Mimeograph and Hectograph.—The methods of the mimeograph and hectograph are quite generally understood but may be mentioned here in connection with reproducing charts.

The mimeograph is probably the most widely known of the stencil duplicators. In this process a stencil is made through which a special ink passes to the paper. The original drawing is usually made in India ink on paper, and then traced on the stencil over a specially prepared table with a ground-glass top under which an electric light is mounted. Once the stencil is made, any ordinary demand for duplicates can be met easily and quickly. The stencil can be removed from the machine and used, when it is wanted, several times before it is destroyed.

In using the hectograph method, the chart is drawn on paper with special hectograph ink. This chart is transferred to a gelatine pad simply by moistening the surface of the pad and placing the drawing face down upon it. Then papers placed in contact with the pad receive duplicates of the original. Colored hectograph drawing inks are available. This method is simple and inexpensive and is very satisfactory when less than one

[1] Exhibit 221*A* is an example of the kind of reproduction that can be made with a "line cut," while Exhibit 222 is an example of the kind that can be made by the half-tone process.

hundred copies are wanted. The original, however, can be used only once if the pad is charged to make many copies. Probably the most widely used machine of this type is the "Ditto."[1]

In using either the mimeograph or the hectograph, typewritten material may be duplicated on the same sheet with the charts.

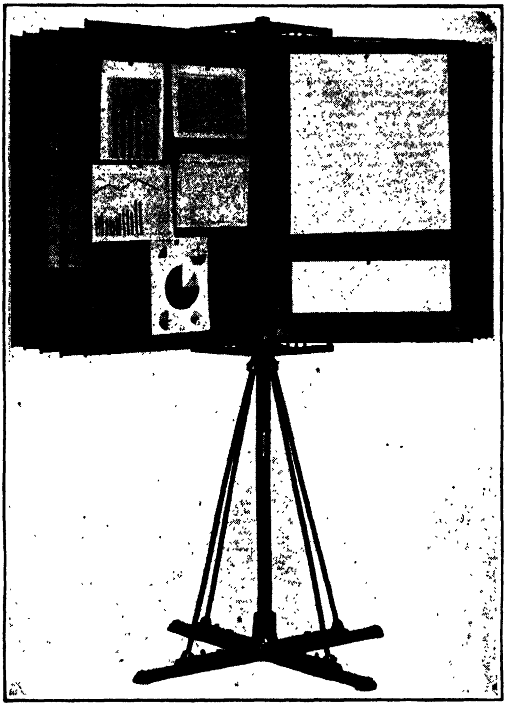

EXHIBIT 223.—Fixture with swinging leaves for displaying charts. (*Courtesy of Multiplex Display Fixture Co.*)

Other similar methods may be suggested by reference to the advertisements in business magazines.

Lantern Display.—It is often desirable to present a chart or table to groups in such a manner that various features may be pointed out directly and discussed. For this purpose, wall charts serve very well but they are expensive to make. Lantern slides are much cheaper and are very satisfactory for showing charts to a large number of persons. If the group is small, however,

[1] Made by Ditto, Inc., Chicago.

the card records themselves may be presented by means of the reflectoscope. This method has a distinct advantage over slides, for small groups, as it is much less expensive and may be kept up to date much more easily. If records are kept on 4- by 6-inch or 4- by 12-inch cards, the very latest information or data on file may be reflected on the screen for use in meetings such as those of department heads or boards of directors.

Display Fixtures.—If charts are to be of the greatest service, it is important to have them readily available. Current charts should not be hidden away in the files but should be where they may be referred to instantly and with very little effort. One of the best methods of handling such charts is to mount them on display fixtures such as the one illustrated in Exhibit 223. There are many makes and types of these fixtures, and many suggestions for display will be found by consulting the catalogues of the various fixture companies that advertise in business magazines.

APPENDIX

TECHNIQUE OF DRAWING AND LETTERING STATISTICAL CHARTS[1]

The actual drawing of statistical charts is not an art which should be studied only by the comparatively few who intend to become professional makers of them, but it should be understood by all interested in business statistics, because of the training that it gives in quick and accurate reading of data presented in the graphic language. A knowledge of statistical drafting is also helpful in a number of other ways. It may be the means of securing a position, since it is a definite attainment which the employer can put to immediate practical use, and the applicant can show in advance what he can do. Later, after the statistical draftsman has worked his way up in the department, his knowledge of drafting will be very helpful to him in supervising chart making, and will better enable him to judge the quality of chart work whenever and wherever he may be called upon to give an opinion.

From the point of view of making charts of business statistics, this appendix chapter on the technique of chart making is designed to cover (1) a description of drawing instruments and materials and their uses, and (2) hand and mechanical lettering.

DRAWING EQUIPMENT AND ITS USE

In laying out and drawing charts, various kinds of drafting instruments are used. The ability to use these instruments in a correct and efficient manner is quite necessary if one is to be an expert chart maker or a good critic of chart work.

In the selection of drawing equipment, one should obtain the best that can be afforded, for success in charting depends to a considerable extent upon the quality of the instruments and materials used. Inferior equipment is an annoyance and continually distracts attention from the actual work in progress, while good equipment will, with practice, leave the mind free from attention to its use and add an environment which is conducive to high-grade work.

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[1] Credit should be given to Thomas E. French and his "Engineering Drawing" for many of the ideas on instruments and lettering presented in this appendix.

The two requirements in the use of instruments are *accuracy* and *speed*, and in drawing charts of business statistics one is as important as the other, and neither is worth much alone. Accuracy must be insisted upon at the outset, and speed with greater accuracy comes with practice. It is natural, however, for the beginner to waste a considerable amount of time because of the strangeness of his instruments and overcarefulness, so the requirement of speed must receive early attention.

The following list includes the instruments and materials necessary for ordinary work in graphic presentation of business statistics.

List of Instruments and Materials

Set of drawing instruments in case, including compasses with needle point, pen, pencil, and lengthening bar; dividers; bow instruments; and ruling pens.
Drawing board.
T-square.
45 and 30–60 degree triangles.
French or irregular curves and ship curves.
Statistician's scale (12 inch).
Percentage protractor.
Lettering pens and penholders (assorted sizes).
Lettering guides.
2H and 5H drawing pencils.
Art gum, erasers, and erasing shield.
Thumb tacks.
Paper.
Other equipment which may be included is mentioned in the description. Instrument dealers' and manufacturers' catalogues will furnish additional suggestions.[1]

Charting Paper.—Selection of drawing paper depends upon the use to which it is to be put and the personal taste of the individual, but in general it is best to avoid poor grades. A good paper should be strong, of uniform thickness and surface, should stretch evenly and lie smooth, and allow considerable erasing without spoiling the surface. Cream, buff, and green papers are much easier on the eyes than white paper, but ordinarily white paper should be used if the charts are to be reproduced by any kind of photographic process. For most business graphic work, many of the less expensive papers are quite satisfactory if they take ink and color well and stand a reasonable amount of erasing. Bond and ledger papers are also used to a considerable extent in chart making.[2]

If the chart is to be reproduced by the blueprint or related methods, the drawing is made on translucent tracing cloth or paper. Tracing cloth is usually preferred to tracing paper because it is more satis-

[1] See especially the catalogues of Theo. Alteneder & Sons, Philadelphia; Keuffel & Esser Co., New York; and Eugene Dietzgen Co., New York.

[2] Detailed descriptions of drawing paper with sizes and prices are given in catalogues of dealers in engineers' and draftsmen's supplies.

factory to work upon, more translucent, and more durable. It is a fine cloth treated with a starch preparation. One side is glossy and the other is dull. Either may be used but most draftsmen prefer the dull side. Care should be taken to protect tracing cloth from moisture as the starch preparation is highly soluble in water.[1]

Pencils.—Lead pencils are graded according to their hardness which is expressed by numbers with the letter H.[2] Personal preferences vary, but for general use in statistical charting a 3H, 4H, or 5H pencil, sharpened to a long conical point, is the most satisfactory. A softer pencil, 2H or H, should be provided also for lettering. The chisel or wedge point is preferred by some for hard pencils, as it does not wear away so fast as the conical point. Pencils may be sharpened to long conical and

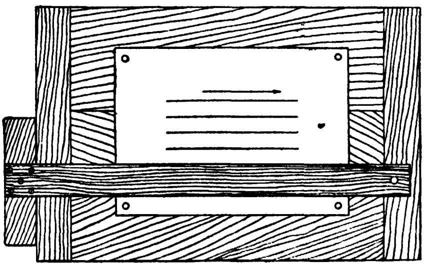

EXHIBIT 224.—Use of drawing board and T-square.

wedge points by removing the wood with a knife and shaping the lead on sandpaper. Special pencil sharpeners are also provided for the draftsmen who care to use them.

The Drawing Board.—Drawing boards and drawing table tops are usually made of well-seasoned, clear white pine cleated at the ends to prevent warping (Exhibit 224). The ends should be perfectly straight in order that the T-square can be used accurately. Drawing boards of 4, 5, or more feet in length are often made in the form of a table. An adjustable drawing table is illustrated in Exhibit 225.

Paper is fastened to the drawing board by means of thumb tacks. It is cut larger than the drawing to be made and should be placed on the board squarely, with the aid of the T-square, and the tacks should be pushed in up to the head so that the T-square will not be obstructed.

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[1] See p. 206 for further information on blueprinting.

[2] Pencils commonly used are harder as the number increases, and are graded from H to 9H. Very soft pencils are graded by their softness which is designated by the letter B and a number which increases as the lead becomes softer. Thus (according to Keuffel & Esser Co.) from soft to hard they are 6B, 5B, 4B, 3B, BB, B, F, HB, H, HH, 3H, 4H, 5H, 6H, 7H, 8H, and 9H. Different companies, however, vary in the use of this system.

The T-Square.—T-squares are usually made of hard wood, the better grades having the edges of the blades lined with ebony or celluloid. They are used principally for drawing parallel lines horizontally and for keeping the triangle in position for drawing other lines. Horizontal lines should always be drawn from left to right, and the T-square should

EXHIBIT 225.—Adjustable drawing table. (*Courtesy of Keuffel & Esser Co.*)

always be used from the left edge of the drawing board (Exhibit 224), as the edges are seldom parallel or at right angles to each other.[1]

Triangles.—Triangles are made of various substances, but transparent celluloid is the most popular. Vertical and sloping lines are drawn by

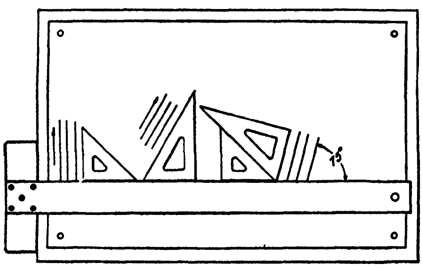

EXHIBIT 226.—Use of triangles.

setting the triangle against the T-square and drawing from bottom to top, as illustrated in Exhibit 226. Various sizes are made, but a 6- or 7-inch triangle will be found convenient for most chart work, especially when 8½- by 11-inch paper is used. As celluloid triangles warp easily, care should be taken that they lie flat or are hung up when not in use.

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[1] This would be reversed for a left-handed man, who would draw from right to left and use the T-square on the right edge of the board.

Irregular Curves and Flexible Rulers.—French or irregular curves and ship curves are made of several kinds of material, but transparent celluloid is the most satisfactory. They are laid out in combinations of spirals, ellipses, and other curves such as will closely approximate those commonly used in practice. A smoothed curve is drawn on a statistical chart by first locating all the points in the series, and then so placing the irregular curve that it coincides with several of them. It is shifted from one position to another as the curve is drawn. It is usually best to sketch through the points a freehand pencil curve that is satisfactory to the eye, and then apply the irregular curve, selecting a part that will

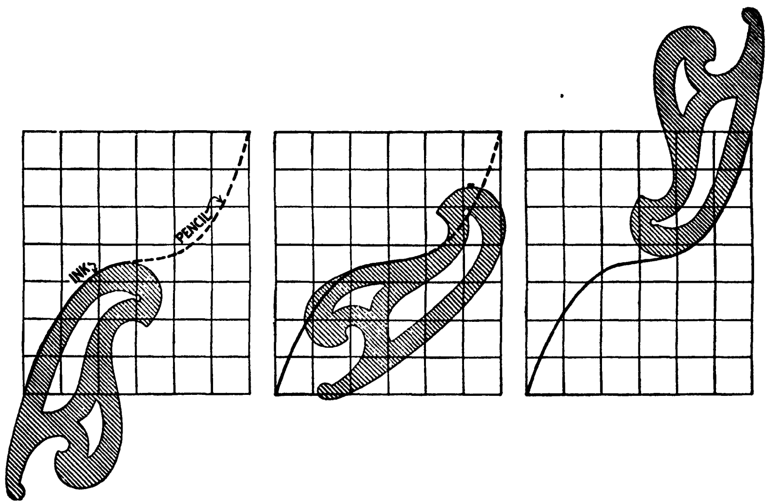


EXHIBIT 227.—Use of French curve. (See catalogues of instrument houses for illustrations of other curves.)

most nearly fit a portion of the line (Exhibit 227). It is very important that the draftsman indicate in advance on his chart the points where the curvature changes. He should then be careful to reverse his curve at such points. Also it is important to select the right type of curve for the problem at hand and to shift this curve so as to get a long fit rather than a short one. When a fit has been obtained, the curve should be used only in its central portion, or portion of best fit, and then it should be shifted and a new fit obtained.

Flexible rulers consist of a ruling edge combined with a soft bar of lead which holds any curve to which the ruler is bent. They are used in much the same manner as irregular curves. They can be adjusted, however, to fit a much larger portion of the curve that is being drawn.

Sometimes, as in drawing a very irregular smoothed curve, time will be saved by making a special templet out of thin cardboard. This is

done first by sketching a freehand curve in pencil through the plotted points. Then the curve is transferred by carbon paper to a sheet of light cardboard. The cardboard is then cut as smoothly as possible along the curve with a pair of sharp scissors, and the curve is further smoothed with a file or sandpaper. When the thin cardboard is backed with a piece of heavy cardboard so that it allows the curve edge of the templet to project slightly, it is ready to be used as a guide in inking in the curve on the finished chart. Guides of this type were used in inking in the curves in Exhibit 171 on page 161.

Scales.—Statistician's scales are made especially for laying out statistical charts. The one shown in Exhibit 228 has four different scales: (1) one in ordinary English rule divisions—sixteenths of an

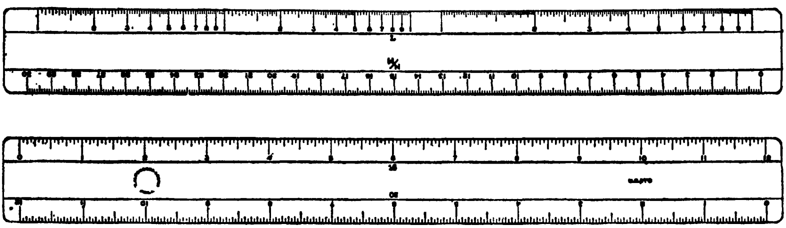


EXHIBIT 228.—A 12-inch statistician's scale. (*Courtesy of Theo. Alteneder & Sons.*)

inch; (2) one in decimal parts of an inch; (3) one in centimeters; and (4) one logarithmic. Thus, a scale is furnished for making logarithmic graduations, and there is a choice of three units for making various measurements, laying off spaces, or dividing a given distance into a certain number of equal parts. The scale illustrated has the advantage of having all the scales necessary for chart making in a small and compact form, but it must be tilted in order to bring the edge of the scale into contact with the paper. This scale is made of plain boxwood or boxwood with white celluloid edges.[1]

There are two kinds of engineer's scales in common use—known as the "engineer's scale" of decimal parts and the "architect's scale" of proportional feet and inches. They are made in triangular form and also in sets of flat shapes. In chart making these scales are quite widely used, especially by those who have had engineering training.

After the proportions and size of the chart have been determined, a convenient method of using a scale in dividing a given space into any number of equal parts is shown in Exhibit 229*A*. The same principle

[1] Scale Number 2735 of Theo. Alteneder & Sons, Philadelphia, Pa. A triangular scale with logarithmic graduations is made by Keuffel & Esser, New York. Special scales for statisticians can be made to order by scale manufacturers.

applies to the use of the logarithmic scale (Exhibit 229*B*). If in such a case the logarithmic scale is too short, a line twice the length of the scale can be laid out between the limiting lines, and each logarithmic division doubled as it is laid out along this line.

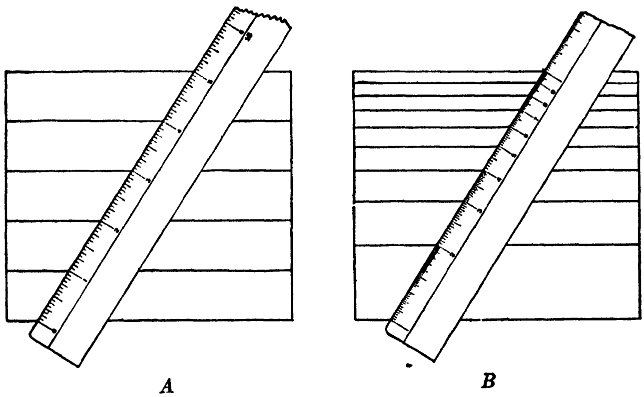


EXHIBIT 229.—*A*, Arithmetic division of a given space. *B*, Logarithmic division of a given space.

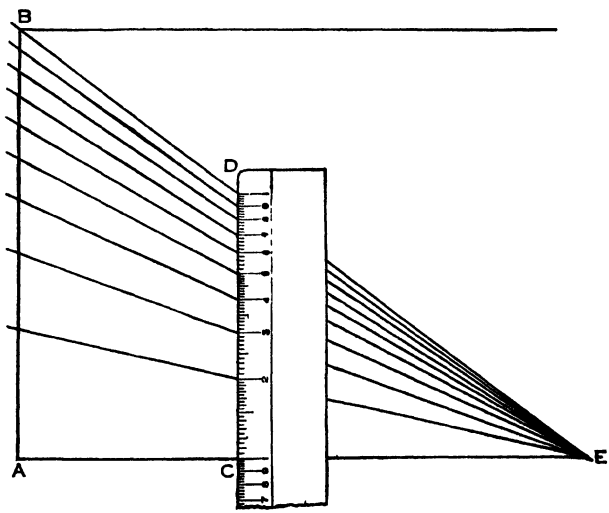


EXHIBIT 230.—Division of a given line.

Another method which is often convenient in using a logarithmic scale, and which may be used also with an arithmetic scale, is that shown in Exhibit 230. Suppose that it is desired to divide the distance *AB* logarithmically from 1 to 10 with the scale *CD*. Place the scale *CD* a reasonable distance from the line *AB* and perfectly parallel with

it. Locate as many points along *CD* as are wanted on *AB*. Draw the lines *BD* and *AC* and produce them until they intersect at *E*. Then lines drawn from *E* through the points laid off between *C* and *D* will locate the logarithmic graduations on the line *AB*. This method, of course, can be used for logarithmic spacing of a distance either greater or less than the length of the scale used.

A convenient method for dividing space as, for instance, in laying out a bar chart where the spaces between the bars are not the same as the widths of the bars, is shown in Exhibit 231. With the dividers or scale simply lay out the spaces in convenient units of the right proportions on a strip of paper somewhat longer than the space to be divided, and use the strip as illustrated in Exhibit 231.

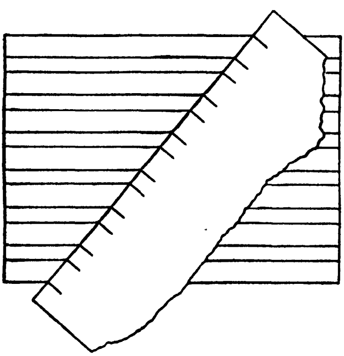


EXHIBIT 231.—Irregular division of a line or space.

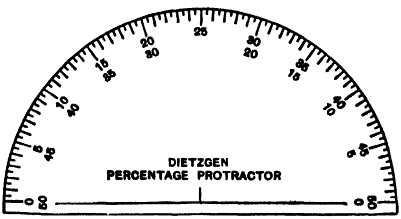


EXHIBIT 232.—A percentage protractor. *(Courtesy of Eugene Dietzgen Company.)*

Protractors.—A protractor (Exhibit 232) is a necessity in laying out "pie charts." The common protractor is a semicircle of 180 degrees, but a more convenient form for pie charting is the percentage protractor made on a basis of 100 parts in a circle. With this form the reading can be made directly from the scale in percentage terms, which is easier and quicker than reading proportional degrees.

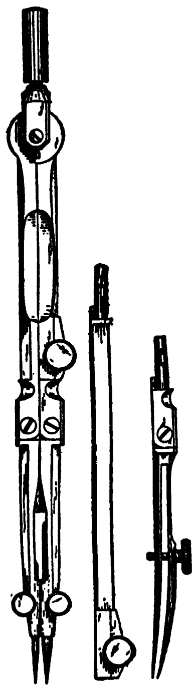
Compasses and Dividers.—Compasses and dividers are usually included in sets of various combinations with a special case. These combinations vary according to the line of work and means of the user, but should contain at least a pair of compasses with pen, pencil, and lengthening bar, a pair of dividers, and a ruling pen. It is advisable to have an instrument case, for it protects the instruments when they are not in use, and it is a convenience when handling the tools, as each one has its special place.

The best instruments are made of German silver and steel, which combine good wearing qualities with ease in keeping clean. The cheaper class is made of brass, which tarnishes easily and is generally unsatisfactory. All modern high-grade compasses and dividers are made with some form of pivot joint, which allows ample movement

of the legs, and at the same time prevents the joint from becoming loose from wearing, which frequently happens with tongue and pin joints.

One leg of the compasses is fitted with a needle point, and the other may be fitted with either a pencil or a pen by means of a shank and socket (Exhibit 233). With most instruments the needle is separate and has a square shoulder, on one or both ends, from which a minute point projects. Before compasses are used, this needle point should be adjusted by attaching the pen and setting the needle a trifle longer than the pen (Exhibit 234). The needle should be kept in this position and the lead, sharpened to a fine wedge or long bevel edge, should be adjusted to it. This will allow the interchanging of pen and pencil without adjusting the needle point.

When a circle is drawn, the compasses, held loosely between the thumb and forefinger, should be inclined slightly in the direction of revolution. Only a light pressure is necessary. In order that both nibs of the pen may touch the paper and only a small center hole be made, it is necessary that the pen and needle be nearly perpendicular to the paper; this is accomplished by bending the legs at the knee joints. For drawing circles larger than the spread of the compasses, the lengthening bar (Exhibit 233*B*) is inserted between the leg socket and the pen or pencil.



A *B* *C*

Exhibit 233. *A*, Compasses. *B*, Lengthening bar. *C*, Pen. (*Courtesy of Theo. Alteneder & Sons.*)

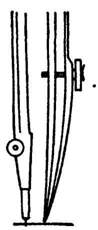


Exhibit 234.—Adjustment of the needle point.

Exhibit 235.—Hair-spring dividers. (*Courtesy of Theo. Alteneder & Sons.*)

Dividers (Exhibit 235) are used for laying off distances, either from measurements or from other parts of the drawing, and for dividing lines into equal parts. When laying off equal distances the dividers should be turned in opposite directions each time, in order that the instrument can be operated readily with one hand.

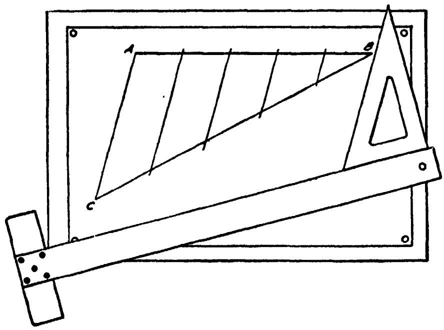


EXHIBIT 236.—Division of a line with dividers.

"Hair-spring" dividers are equipped with a screw adjustment in one leg for making fine settings. Combination instruments are made which may be used as either compasses or dividers simply by interchanging the pen or pencil point with a needle point.

A convenient method for dividing a line into any number of parts is illustrated by Exhibit 236. To divide line *AB* into, for instance, five equal parts, draw any line *BC*, and on it step off with the dividers

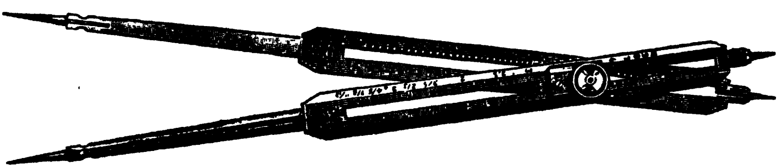


EXHIBIT 237.—Proportional dividers. (*Courtesy of Keuffel & Esser Co.*)

five convenient lengths. From *C*, the last point, draw a line to *A*. Then draw lines parallel to *AC* through the points on *BC*, intersecting the line *AB*, which will be divided thereby into five equal parts. Triangle and T-square may be used as illustrated for drawing parallel lines.

Another form of dividers, ordinarily not included in sets, is the proportional dividers (Exhibit 237) which are used for enlarging or reducing charts in any proportion. The subdivisions marked "lines" are for linear proportions, while those marked "circles" are for dividing into any number of equal parts a circle whose diameter is measured by the large end.

The bow instruments are used as adjuncts to the compasses and dividers and have their special field of usefulness in making circles and spaces which are so small that it is difficult to make them with the large instruments. A set of three spring bow instruments includes a pair of bow dividers or spacers, a bow pencil, and a bow pen (Exhibit 238). Since it holds its adjustment, the bow pen is almost indispensable in duplicating a large number of small circles.

The drop pen (Exhibit 239), with which small circles can be made much faster than with the ordinary bow pen, is of particular convenience in making statistical maps in which small circles or large dots are used. It is held with the needle point stationary and the pen revolving around it.

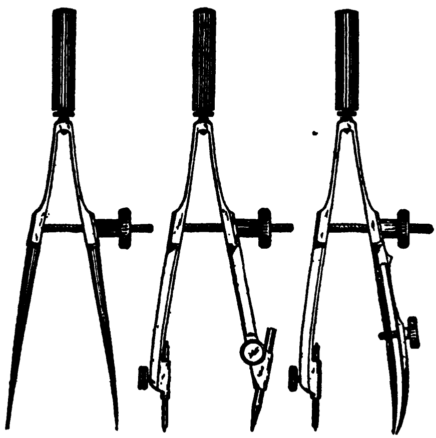


EXHIBIT 238.—Bow instruments. (*Courtesy of Theo. Alteneder & Sons.*)

Ruling Pens.—Ruling pens are made in a number of forms, but all consist of a handle bearing two spring nibs which have a thumbscrew adjustment for regulating the width of line (Exhibit 240). Some forms (Exhibit 240*B*) have a device which allows the blades to be opened for cleaning, independently of the thumbscrew. For large work a very valuable pen is the form known as the detail pen (Exhibit 240*C*).

Such forms of special pens as the dotting pen, curve pen, and railroad pen are of value occasionally in drawing curves, though they are not required in ordinary work. The dotting pen may be used when making several distinct curves on a chart; it will make several different kinds of dotted or dashed lines successfully when carefully handled. The curve pen is made with a swivel for curve drawing, and the railroad pen is used for making double lines.

The ruling pen is filled by placing the ink between the nibs with a common steel pen or with the quill filler or dropper that is provided

with most drawing inks. The ink should fill the pen to a height of from $\frac{3}{16}$ to $\frac{1}{4}$ inch depending upon the size of the pen; if too much is used, its weight will cause it to run out upon the drawing. The width of the line is varied by changing the distance between the pen points with the thumbscrew.

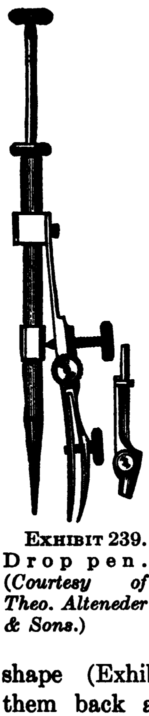


EXHIBIT 239. Drop pen. (*Courtesy of Theo. Alteneder & Sons.*)

The ruling pen is never used freehand, but always with a guiding edge. It should be held with the blades parallel to the edge, the handle slightly inclined in the direction of motion, and in a plane which passes through the line perpendicularly to the paper (Exhibit 241).

Ink will dry rapidly in the point of the pen when it is adjusted for fine lines, and may refuse to flow after the pen has remained idle for a few seconds. If gently pinching the nibs or touching the point on the back of the hand does not start the flow, the pen should be cleaned and filled with fresh ink. When one is through using a pen it should be wiped clean, because a dirty pen will not draw clean-cut lines and ink left in the pen will corrode the steel blades (Exhibit 242).

Inked lines may be imperfect because of the fault of the pen, the ink, the paper, or the draftsman. Exhibit 243 shows several common faults for which the draftsman is to blame.

If a pen has become worn or is too sharp or uneven, it can be put in proper condition by grinding on an oilstone. The points should be given first the proper shape (Exhibit 244) by screwing them together and drawing them back and forth over the stone. Then the pen should be opened slightly and the *outside* of each blade ground by giving the pen a rocking motion at a small angle with the stone. One must be

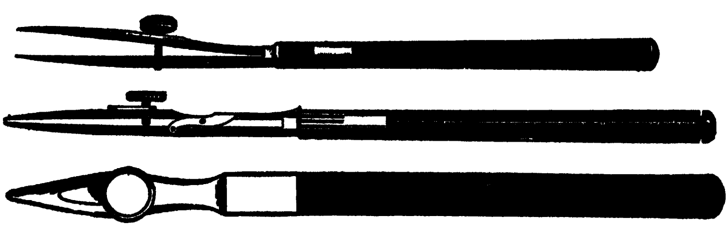


EXHIBIT 240.—Ruling pens. (*Courtesy of Theo. Alteneder & Sons.*)

careful not to alter the shape of the points or to grind them too sharp. They should be sharp enough to draw a fine hair line, but not sharp enough to cut the paper. If a burr is formed on the inside, it may be

removed by touching it lightly upon the stone, but the inside of the blades should never be ground.

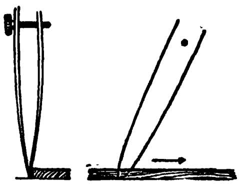


EXHIBIT 241.—Correct position for using ruling pen.

EXHIBIT 242.—Pen blade corroded by ink.

Ink.—Waterproof India ink is generally used for chart making, as it leaves a permanent black line. It is composed of finely divided carbon held in suspension, with shellac added to make it waterproof. Non-waterproof ink flows more freely, but is not popular because it smudges

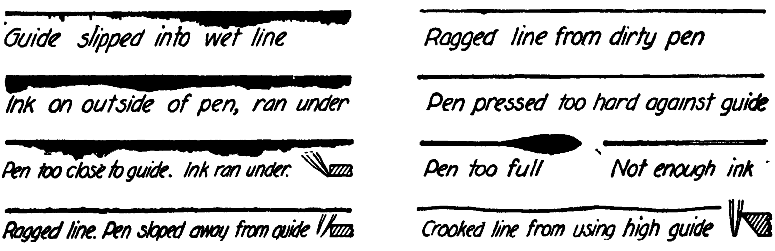


EXHIBIT 243.—Beginners' common faults in ruling lines.

very easily. A blotter should never be used on drawing ink, as it removes part of the layer of carbon and leaves a very weak line.

Black drawing ink can be secured in sticks, but the more convenient liquid form, furnished in bottles equipped with a quill for filling the pen, is commonly used. A handy ink bottle holder and pen filler, which can be operated readily with one hand, is illustrated in Exhibit 245.

EXHIBIT 244.—Correct shape for ruling pen blades.

When not in use, ink bottles should be kept tightly corked to keep the ink from thickening and drying up. Drawing ink must not be allowed to freeze, as once frozen it is useless.

Waterproof drawing inks are also made in a variety of colors. As colored inks are thinner than India ink, they flow more readily when used in a ruling pen.

Erasers.—Although but little erasing should be necessary, a good ink eraser should be provided for erasing errors in lines or letters. In using erasers, care must be taken not to damage the paper or the

surrounding work. Erasing shields, usually made of thin metal, are used to protect the work around the erasure, which is made through one of the slits.

Exhibit 245.—A handy ink-bottle holder and pen filler. (*Courtesy of Theo. Alteneder & Sons.*)

Drawings are cleaned by rubbing them with a piece of art gum or sponge rubber. However, a finished drawing should not be scrubbed all over with art gum or rubber, as it takes the life out of the inked lines.

Tinting.—The usual method of tinting is to apply light washes of water colors with a camel's hair or sable brush. As red sable brushes are not affected by water, they are better for this work than camel's hair brushes which lose their elasticity after being wet. These brushes are commonly fitted with long handles, which may be made more convenient for some work by being cut down to a length of about 7 inches. Three brushes of convenient size for ordinary chart work are shown in Exhibit 246. Brushes should be cleaned after each time they are used, and should not be allowed to stand with the weight of the handle on the hair.

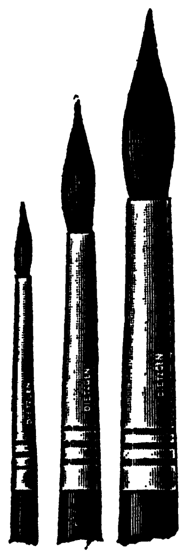


Exhibit 246.—Water color brushes—full size. (*Courtesy of Eugene Dietzgen Co.*)

After the drawing has been inked and cleaned, the part to be tinted should be gone over with a wash of clear water in order that the color may spread evenly. A pool of color is then led with a brush over the surface. After the surface has been covered, the brush should be wiped out and used to pick up the excess color. The tints should be mixed as light washes in shallow dishes. Deep colors can be obtained by repetition of the washes. One should see that the wash is thoroughly mixed and that it is not too thick.

Colored inks may be diluted and used in the same way as water color washes.

Crayons are handy to use and, if they are carefully applied, good effects may be produced. They should be applied very lightly with a flat point and with parallel strokes. Depth of color should be secured by repetition of strokes—not by pressure which makes it impossible to get an even color. Various line or dot effects can be secured by placing the drawing over different kinds of irregular surfaces, such as book bindings, and

then rubbing with crayons. Some crayon work can be evened by smudging with a cloth over the end of the finger, but usually this process takes the life out of the color. Work done in soft crayon may be made permanent by spraying a shellac fixative over it with an atomizer.

Shading.—In cross-hatching shades on bar charts, belt charts, and maps, the mechanically spaced rule, commonly called the *section liner*, is a valuable aid. This is especially true when the system illustrated in Exhibit 210 on page 194 is used. The section liner can be adjusted for various spacings, and the ruling edge is moved from one space to another by pressing a small lever. Only the better grades of section liners are recommended for such shading as that in Exhibit 210, as the spacing devices must be highly accurate and dependable.

A makeshift section liner, which is very satisfactory when skillfully used, can be made with a T-square, a triangle, and a coin which is somewhat smaller than the hole in the triangle. The T-square is fastened to the drawing board with thumb tacks. The triangle is set against the T-square in the usual manner, and the coin is placed in the hole in the triangle to control the distance that it can be moved. With the fingers on the triangle and the thumb on the coin, the coin is moved as far as it will go; then, while the coin is held securely, the triangle is shifted as far as it will go, and so on. For dense cross-hatching, a coin only slightly smaller than the hole in the triangle would be used. For sparser hatching, a smaller coin would be used, which would allow a greater movement of the triangle.

When the shading problem is simply one of differentiating areas—that is, when the different shades themselves do not represent statistical magnitudes, the air-brush method is very satisfactory and efficient. Black or colored inks or washes can be sprayed on a chart very rapidly and smoothly with results not possible in hand work. The air brush is especially useful in making up charts where special artistic effects are desired, such as in connection with window displays or popular advertising exhibits.

White lettering or lines may be made to show through air-brush shading by drawing them in rubber cement before the air-brush work is done. After the shading is completed, the cement is removed, care being taken not to rub the adjoining inked areas too much. Rubber cement can likewise be used to protect areas from ink when other than air-brush methods are used.

Optical Illusions.—In cross-hatching or shading areas it must be recognized that one must contend with certain optical illusions. Occasionally these illusions spoil the effect of the entire chart. Some of the most common are illustrated in Exhibit 247. Note that in *A* the part of the bar shaded with horizontal lines appears to be narrower than the

parts shaded with vertical lines. In *B* the part of the bar shaded in black appears to be narrower than the white parts. In both *A* and *B* the sides of the bars are perfectly straight and parallel. In *C* the first and second bars appear to be closer together at the ends than in the middle, and the second and third appear to be farthest apart at the ends while, as a matter of fact, the three bars are perfectly parallel.

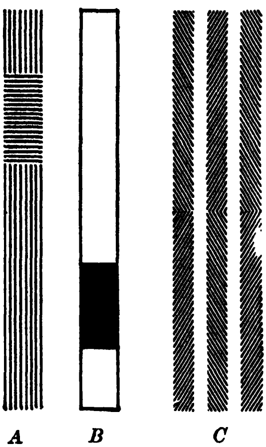


EXHIBIT 247.—Optical illusions.

Copying Devices.—A convenient device for copying charts on heavy paper is the glass copying board or table. It is so made that natural or artificial light passes through the glass from below and the opaque lines and letters can be distinguished easily from the more translucent parts. The better forms consist of a plate-glass top over a ground glass which evenly distributes the light furnished from underneath by electric lamps mounted over a bright reflector (Exhibit 248). This device is valuable also in comparing different charts by superimposing one over the other. As an emergency substitute, a chart may be clipped to the drawing paper and held up to the light against a window.

For transferring the outline of a chart, carbon paper can be used provided the original is not too stiff. The chart and drawing paper, with the carbon sheet between, are so tacked down that one will not move upon the other, and the lines are traced with a small round point.

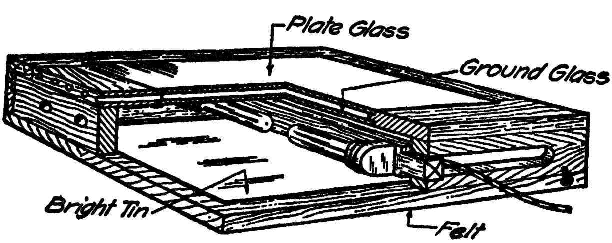


EXHIBIT 248.—Glass board for copying and comparing charts. *(From "Engineering Drawing" by Thomas E. French.)*

A method similar to this is transferring by pricking, in which the chart is laid over the paper and pricked through with a needle point, turning the chart back frequently and connecting points.

Charts can be reduced or enlarged in any proportion by means of the pantograph. It consists of a parallelogram of four bars having the tracer, pencil, and pivot in a straight line. Any adjustment of the bars

in conformity with this arrangement will copy in true proportion. A simple wooden form is inexpensive and accurate enough for most chart drawing, but better work can be done with the finely constructed, suspended pantograph.

Mounting and Adhesives.—There is no single method of mounting that can be used for all cases. Different purposes and materials require different adhesives and handling.

A very satisfactory adhesive for temporary mounting in general chart work is liquid rubber cement. It does not warp the paper, and any excess may be rubbed off the chart with the finger or an eraser, leaving the paper clean. Paper mounted with this cement is held securely, but may be pulled off without tearing, as the cement is not so strong as paper. As different kinds of cement vary widely, care should be taken, when rubber cement is purchased for use on paper, to select that which sticks well and is light in color. Special rubber cements for mounting paper may be obtained from stationery dealers. Some kinds of cement provided for repairing automobile inner tubes are also satisfactory for mounting paper. These cements may be spread with the finger or with a brush and should be applied freely over the entire surface of the paper, which then should be pressed firmly into place. A stronger joint can be made, however, by coating both surfaces and letting them dry a few minutes before pressing them together. Where great strength is desired, as in joining paper with a small lap, rubber cements are not practical, and for permanent mounting they are unsatisfactory, as they lose their adhesive qualities with age.

If paper is stretched on a drawing board and the edges fastened with glue, another paper can be mounted on it with thin photographic paste or liquid glue without warping. The papers must, however, be allowed to dry thoroughly before being cut from the board. If loose papers are stuck together with paste or glue, they will warp, but sometimes they can be smoothed out reasonably well with a hot iron.

There are two common methods of mounting paper on cloth—hot mounting and cold mounting. Hot mounting can be done much more rapidly than cold mounting and for this reason is generally used. To hot mount a chart, first stretch the cloth tightly over a cloth-covered board and fasten it securely with tacks. Then cover the back of the chart with a thin hot photographic paste or liquid glue, brushing from the center outward. Have a hot iron ready and press the chart down upon the cloth, working rapidly from the center toward the edges. As soon as the edges are stuck, remove the tacks and raise the cloth to let out the steam. Iron on the chart side until dry. If the iron is used on the cloth side, the steam formed will cause blisters. Cold mounting is done in the same way, except that cold paste and a photographic print roller are used, instead of hot paste and an iron, and the chart is allowed to

dry thoroughly before the tacks are removed. Paste and glue work well in both methods, but glue breaks the more easily if the chart is folded.

Ordinary adhesive tape is very convenient for such purposes as temporarily mounting a chart on a wall where tacks cannot be used. Some draftsmen use it to fasten paper to the drawing board. Many other uses for adhesive tape will suggest themselves if a roll is kept readily available.

LETTERING

Lettering is closely related to drawing but is not part of it. It is a separate and distinct subject, covering the making and use of letters of various accepted styles and sizes. Letters may be made either by hand or mechanically. Hand lettering is the process of making them by the freehand single stroke method or by the method of drawing them with the aid of drafting instruments. Mechanical lettering involves the use of mechanical equipment such as the printing press or the typewriter. As far as appearance is concerned, there is no part of a chart more important than the lettering. A good drawing may be ruined by poor or careless lettering, not only in appearance but in its effect, as any evidence of carelessness or lack of skill tends to destroy confidence in the work.

OLD ROMAN

Roman lower case

MODERN ROMAN

Gothic

COMMERCIAL GOTHIC

EXHIBIT 249.—A few of the different styles of letters commonly used by printers. (Only the plainer types are used on statistical charts.)

Two fundamental requirements, which any method of lettering used in graphic presentation of business statistics must possess, are *legibility* and *speed*. If it has these qualities, almost any consistent style of plain, neat lettering is acceptable in the ordinary course of chart work. Ornate lettering has its place among artists and craftsmen but is not practical or appropriate for business charts.

Styles of Letters.—The most common styles of letters used in charting are the "Commercial Gothic," and the "Modern Roman" or its modifications (Exhibit 249). In mechanical lettering both styles are used. In hand lettering the style used chiefly is a modified form of Commercial Gothic.

Hand Lettering.—There are two general divisions of hand lettering—freehand single stroke or written letters, and drawn or built-up letters. Small Commercial Gothic are made by the freehand single stroke method, but the large size are usually drawn with the aid of drawing instruments. Both large and small Roman are commonly made in outline and filled in.

Lettering Instruments.—Many different kinds of steel writing pens are used for lettering. Exhibit 250 shows several pens which are widely used for making letters under ¼ inch high. The size of pen to be used depends upon the width of line desired for the lettering. The pen should be selected which will make the desired width without spreading the point. Fine pens should not have points so sharp that they will

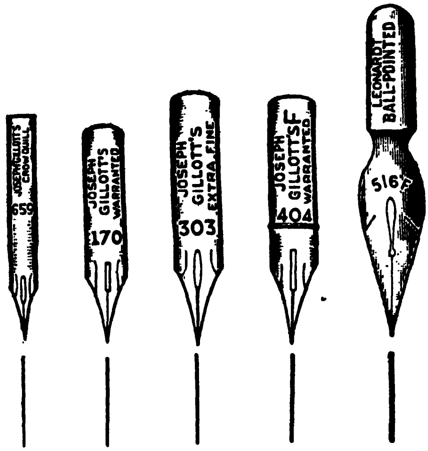


EXHIBIT 250.—Lettering pens. (*Courtesy of Eugene Dietzgen Co.*)

scratch the paper, and for this reason lithographic pens are not well adapted for chart lettering. New pens are covered with a very thin film of oil which should be removed before they are used, by wetting and wiping dry. Steel pens may be inked by dipping them into the bottle, but many consider it better to fill them with the ink-bottle quill or a dropper. If too much ink is used, it has a tendency to run where strokes are joined in the letters (Exhibit 251). Lettering pens should be given as good care as other instruments, for a well-broken-in pen is more valuable than a new one. The ordinary cork grip penholder is commonly used, though various other kinds are furnished to suit personal preferences.

EXHIBIT 251.—Too much ink.

A very popular form of special pen for making letters from ⅛ to 2½ inches high is the Payzant pen (Exhibit 252). Uniform lines can be made easily with this pen, as it is constructed to produce the same width and density of line, no matter in what direction the pen is moved over the paper. It is obtainable in eleven sizes, which make lines from 0.012 to 0.2 of an inch wide, as illustrated in Exhibit 252. The Payzant pen is filled by placing the ink in the reservoir with a quill or dropper. It should never be dipped into the ink. When the pen becomes clogged,

the flow can be started by inserting a piece of stiff paper between the nibs as illustrated in Exhibit 252 (bottom pen). Strokes made with a Payzant pen have rounded ends, and if a better finish is desired, sharp corners may be added with a fine steel pen.

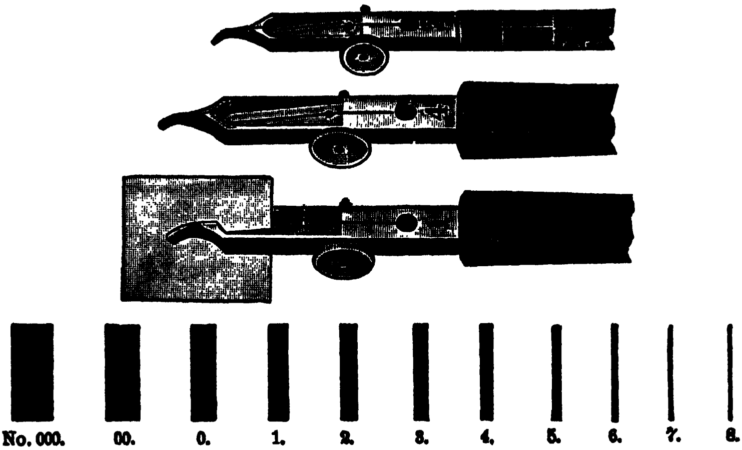


EXHIBIT 252.—Payzant lettering pens. (*Courtesy of Keuffel & Esser Co.*)

Letters larger than 2½ inches high, such as those used in titles of wall charts, are best made by outlining with drawing instruments and filling in, but fair letters may be made much more quickly by using a brush or a pen such as the automatic marking pen (Exhibit 253). When the automatic marking pen is used, care should be taken to see that the letters are correctly shaped. This pen is made in various sizes for lines from 1/16 to ½ inch in width. It should always be filled with a quill or dropper and never by dipping it into the ink.

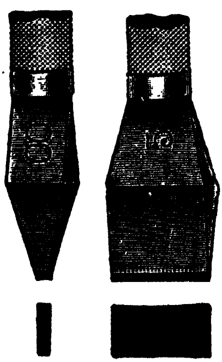


EXHIBIT 253.—Automatic marking pens. (*Courtesy of Eugene Dietzgen Co.*)

A convenience for making guide lines is the Braddock lettering triangle. The pencil point is inserted through one of the holes, and the line is drawn by sliding the triangle along the T-square or other straight edge. The numbers of the columns of holes express the heights of capitals in thirty-seconds of an inch. Vertical or slanting guide lines may also be drawn with this instrument by using it as an ordinary triangle. Several other lettering triangles similar to the Braddock are made by instrument manufacturers.

After one has studied lettering and knows what sizes to use and how the letters and lines should be spaced, the lettering guide illustrated in

Exhibit 254 is a very satisfactory aid in making accurate forms.[1] The lettering on the chart in Exhibit 6 (page 13) is an example of the work that can be done with the aid of this device. A good pen to use with all except the smallest guides is the Payzant illustrated in Exhibit 252. The special pen shown in Exhibit 254, however, is preferred by many and is essential if one is using the smaller guides.

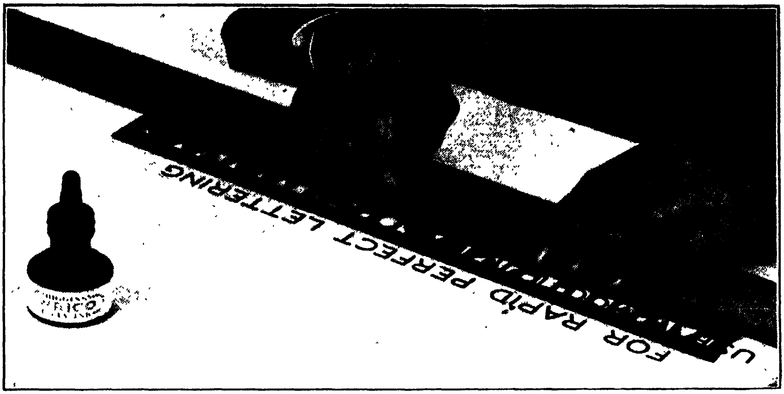


EXHIBIT 254.—Wrico lettering guide. (*Courtesy of Wood-Regan Instrument Co.*)

General Proportions.—There is no standard proportional relation of width to height in letters. When space for lettering is short, **COMPRESSED LETTERS**, which are very narrow in width as compared to height, are used. When it is desired to spread a short title over a long space, **EXTENDED LETTERS**, which are very wide in proportion to their height, are used. The most legible and stable appearance of capitals is secured by making the width about seven-eighths of the height (except I, M, and W). Letters also vary in the thickness of stroke in proportion to height. Those with heavy strokes are known as "**bold face**" and those with narrow strokes as "light face" letters.

In making letters one must contend with certain optical illusions. Round letters, such as C, O, or Q, will appear to be smaller than square letters, such as E, F, or Z, if they are made exactly the same in height. This is true also of letters such as A and V which would appear to be too short at the pointed ends. In order to give the appearance of equal

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[1] The guide illustrated in Exhibit 254 is manufactured by Wood-Regan Instrument Co., 154 Nassau Street, New York, and distributed by Eugene Dietzgen Co., New York. Descriptions of other guides may be found in the catalogues of drafting instrument manufacturers and distributors.

heights, the round or pointed ends of letters should be extended a little over the guide lines which limit the letters that have square ends. If letters such as E, H, or S are to look well-proportioned, their top and bottom halves should appear to be equal. This effect can be secured only by making the letters slightly narrower at the top than at the bottom and drawing the center lines slightly above the middle. The illusion referred to will be quite evident if any printed page is turned upside down and the letter S observed. The point must be emphasized, however, that the corrections for these illusions are very small and that overdoing them is worse than overlooking them.

Freehand Single Stroke Lettering.—The term "single stroke" means that the width of a single stroke is the width of line used in making the letter, and "freehand" means that no guiding instruments are employed and the pen is used as in writing. Freehand is the lettering most commonly used on statistical charts, and a knowledge of its construction is very necessary for anyone who makes them. The ability to letter well can be acquired by anyone who has normal control of his fingers, who will practice faithfully, and who will observe carefully the shapes of the letters. There is no direct relation between one's lettering and his handwriting. Many good letterers write very poorly.

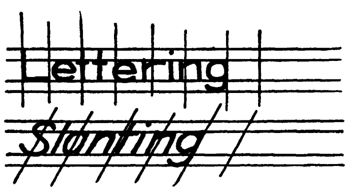

Exhibit 255.—Guide lines for lettering.

In lettering, the pen should be held lightly, as in writing, and the strokes drawn with a uniform motion, with a light, even pressure on the paper. The stroke should be complete in itself and should not be sketched in or retraced. The nibs of the pen should not spread. If a heavier line is wanted, a coarser pen should be used. Horizontal guide lines should always be penciled in as an aid in making the tops and bottoms of the letters even and straight (Exhibit 255). Other lines may be ruled in at random to aid in making strokes vertically or at a certain slant (Exhibit 255). The letters may be made directly in ink, but penciling is necessary in composition or laying out symmetrical titles. Vertical strokes are always made downward, and horizontal strokes from left to right.

Vertical Single Stroke Capitals.—The forms of these letters must be very carefully observed and should be made exactly as shown in Exhibit 256. The letters are all made the same in width except M, W, and I. The figures (excepting 1) may be made the same in width as the capital letters, but usually the effect is better if they are made slightly narrower. In height the figures are made the same as the

capital letters. Cross lines are drawn slightly above the center, except in A, G, 4, and 9, in which they are a little below. The similarity in the proportions of the letters B, E, F, H, J, L, P, R, and U is brought out by Exhibit 257, which shows how they would look if superimposed one

EXHIBIT 256.—Vertical single stroke capitals. (*From "Engineering Drawing" by Thomas E. French.*)

upon the other. The sides of the letter M should appear to be parallel, and not sloping as in an inverted W. A very important point to observe in making the letters B, D, P, and R is that the tops and bottoms are

EXHIBIT 257.—B, E, F, H, J, L, P, R, and U superimposed.

EXHIBIT 258.—6, 9, and 0 superimposed.

made mainly of horizontal strokes. The backbones of the 6 and the 9 have the same curve as the cipher. Exhibit 258 shows these three figures superimposed.

EXHIBIT 259.—Vertical lower case. (*From "Engineering Drawing" by Thomas E. French.*)

Vertical Lower Case.—The alphabet of vertical lower case letters is made as shown in Exhibit 259. These letters are all of the same width except the seven letters f, i, j, l, m, t, and w. The bodies are made two-thirds of the height of capitals, with the ascending stems reaching the

cap guide line and the descending stems reaching the same distance below (Exhibit 260). The circle and parts of it combined with straight

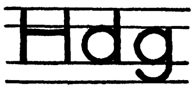

EXHIBIT 260.—Proportions of letters.

EXHIBIT 261.—a, b, c, d, e, g, h, n, o, p, q, r, u, and y superimposed.

lines form the basis of the letters a, b, c, d, e, g, h, n, o, p, q, r, u, and y. These fourteen letters are shown superimposed in Exhibit 261.

EXHIBIT 262.—Inclined capitals. (*From "Engineering Drawing" by Thomas E. French.*)

Inclined Capitals.—The inclined or slanting alphabet shown in Exhibit 262 is easier to make than the vertical, as a slight difference in inclination is not apparent, while a slight tendency from the true vertical is very noticeable. The inclination is generally made from 60 to 70 degrees from the horizontal. Special triangles are made for drawing these slope lines. Particular attention must be given to the inclined *A*, *V*, and *W*, in that a slope line drawn through the point of the letter should bisect the angle made by the two sides of the letter (Exhibit 263). The discussion of sizes and proportions in the preceding paragraph on vertical capitals applies equally well to the inclined capitals.

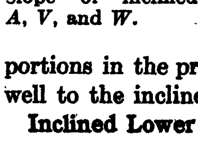

EXHIBIT 263.—Slope of inclined *A*, *V*, and *W*.

Inclined Lower Case.—The inclined lower case letters (Exhibit 264) are founded upon the straight line and the ellipse instead of the circle.

In other respects they are proportioned the same as in the vertical system.

Common Errors to Be Avoided.—Some common errors to be avoided in both capital and lower case letters are illustrated in Exhibit 265.

Large Commercial Gothic.—The preceding description of letter making has covered only the freehand single stroke method of making the Commercial Gothic style of lettering. For large, careful work the letters are outlined and filled in (Exhibit 266). In inking a penciled outline, the ink should not extend outside the pencil line or the letter

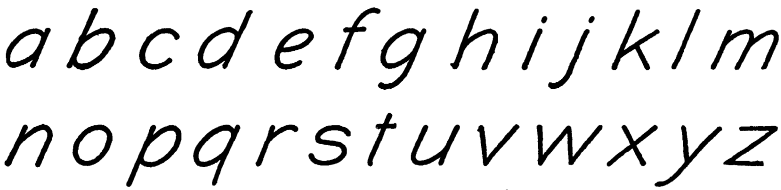

EXHIBIT 264.—Inclined lower case. (*From "Engineering Drawing" by Thomas E. French.*)

will be made heavier than was intended. For lettering that is to be read from a distance, brushes or Payzant and automatic marking pens sometimes are used. Such lettering will stand close inspection when made by a very skillful workman.

Handmade Modern Roman.—The Roman is the most familiar of all styles of lettering, as it and its modifications are generally used in present-day printing. It is, however, a difficult lettering to make, and can be mastered only by very careful attention to the details of forms and proportions.

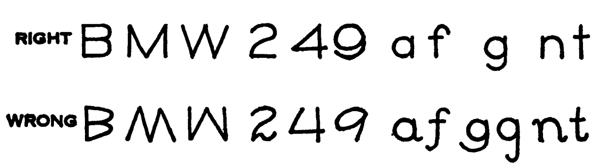

EXHIBIT 265.—Common errors.

Exact rules for the formation of these letters are of little value, as success in making them depends upon the judgment of the draftsman. The two widths of lines, which were originally the upstroke and downstroke of a flexible reed pen, must be arranged in the right order. It is a bad error to shade a Roman letter wrongly. The seriphs at the ends of the strokes are very important parts of a letter, so far as appearance is concerned. A seriph should be joined by a short curve, and not by an angular line. Roman letters are usually made in outline, either

freehand or with instruments, depending upon the size, and then are filled in.[1]

Spacing.—Definite rules for spacing letters cannot be laid down because of the varying proportions of the letters. An appearance of uniform spacing is a matter of artistic judgment, and cannot be gained mechanically. If letters are spaced at equal distances along the guide lines, there will appear to be more space between letters such as AT or OC than between letters with parallel sides such as HN. To give the appearance of equal spacing, the minimum spacing should be used in

Exhibit 266.—Outlined letters for large careful work. (*From "Engineering Drawing" by Thomas E. French.*)

such combinations as AT, AV, LY, or TJ, whereas the maximum spacing should be used in straight side combinations such as HE, NU, or IR. Beginners often make letters too narrow for their height, or space them too far apart. Crowding, however, should also be carefully avoided.

Spacing between words varies with the effect desired, but it should always allow the words to be read easily. In ordinary work the space should appear to be about one to one and one-fourth times the width of the average letter used.

[1] For further information see Thomas E. French and William D. Turnbull, "Lessons in Lettering," McGraw-Hill Book Company, Inc., New York, 1924.

It is very important to allow ample space between lines if the work is to be read easily. This space should be from one-half to one and one-half times the height of the capitals. A crowded appearance will result if insufficient space is left between lines.

In laying out a symmetrical title for a chart it usually is best to balance it on a vertical center line. The letters and spaces in the title are counted and then sketched in pencil, beginning each line with its middle letter at the center line. (Proper allowances should be made, of course, for the effects of M, W, and I.)

Mechanical Lettering.—The term "mechanical lettering" is intended to include all methods in which the letter is produced, not by being built up each time, as in hand lettering, but by some already produced letter form which may be used over and over again. The term is broader than printing (with type) as it includes such methods as stenciling and the use of cut-out letters.

The Drafting-room Printing Press.—Small hand printing presses are made especially for printing on charts and drawings. Any kind of ordinary lettering work can be done much better and faster on these presses than it can be done by the average draftsman. The presses are simple in construction and require relatively little training for efficient operation.

Rubber Stamps.—Rubber stamps can be used for making reasonably good lettering, provided they are used skillfully and with the right kind of ink. A uniform impression can be made with a large stamp by using a simple, inexpensive, rubber type printing press. By using the "Stamp-o-graph process,"[1] opaque, smear-proof letters can be stamped on tracing cloth.

Pasting on Printed Titles.—Very good results can be obtained, in making charts that are to be reproduced by a photographic process, by pasting on titles that have been set up and printed by the ordinary printing process. Several titles can be set up and printed together at a small expense. They can then be cut apart and mounted in the proper positions on the charts. If the edges of the printed strips are painted with Chinese white, they will not show when they are photographed, and the letters will appear to be printed directly on the chart.

Cut-out Letters.—Gummed letters, cut out of black or colored paper, are made for setting up titles. A great many grotesque forms of these letters are on the market, and care should be taken to select forms that look like printed letters. They may be purchased at any well-supplied stationery store.

Typewriter Lettering.—The typewriter can sometimes be used successfully in making certain kinds of small charts (see Exhibit 5 on

[1] Eugene Dietzgen Co., New York.

page 11). There are not, however, enough differences in sizes of type on an ordinary typewriter to give the lettering the right proportions for the various titles, captions, keys, and references usually placed on graphic work. This situation may be improved somewhat by using special machines, such as the "Hammond Multiplex," or by using two or more machines with different sizes of type. Letters can be typed in various colors, without changing the ribbon, by using colored carbon papers.

INDEX

Q

R

T

U